

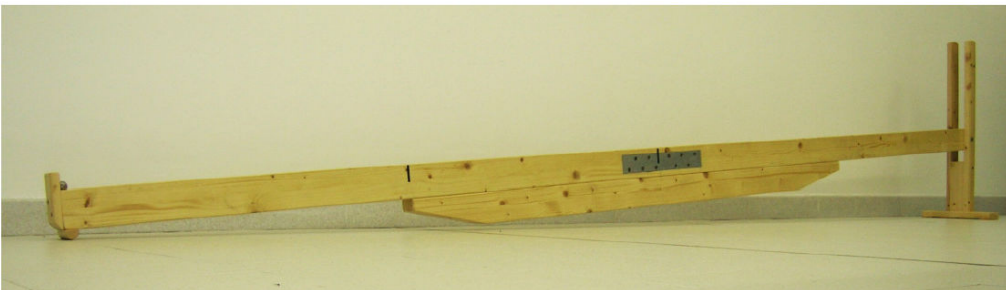


From Galileo to Apollo: Analyzing Gravitational Acceleration with MATLAB

Ball on an Inclined Plane

A ball rolling down a ramp seems like a very simple phenomenon, but we can use our observations of the rolling ball to understand more about the meanings of position, velocity, and acceleration. Galileo Galilei, one of the founders of modern science, accurately measured the effect of gravity on falling bodies using an elegant apparatus: the inclined plane. He began his studies in 1602 and published his findings in his 1638 book, *Two New Sciences*.

Galileo and the Inclined Plane



Reconstruction of Galileo's inclined plane apparatus (Straulino, 2008)

Galileo's rolling ball experiments on the inclined plane were significant because they provided the first experimental confirmation of the laws of uniformly accelerated motion. By carefully timing the descent of a ball along a polished groove using a water clock, Galileo demonstrated that the distances traveled were proportional to the square of the elapsed times, establishing the mathematical relationship that governs acceleration due to gravity. His use of the inclined plane slowed down free fall just enough to make measurement possible, bridging theory and observation in a way that transformed natural philosophy into modern experimental science. The work not only validated his theoretical predictions but also introduced systematic repetition, error awareness, and clear reporting, making it a prototype of the scientific method.

Straulino, S. (2008). Reconstruction of Galileo Galilei's experiment: The inclined plane. *Physics Education*, 43(3), 316–321. <https://doi.org/10.1088/0031-9120/43/3/012>

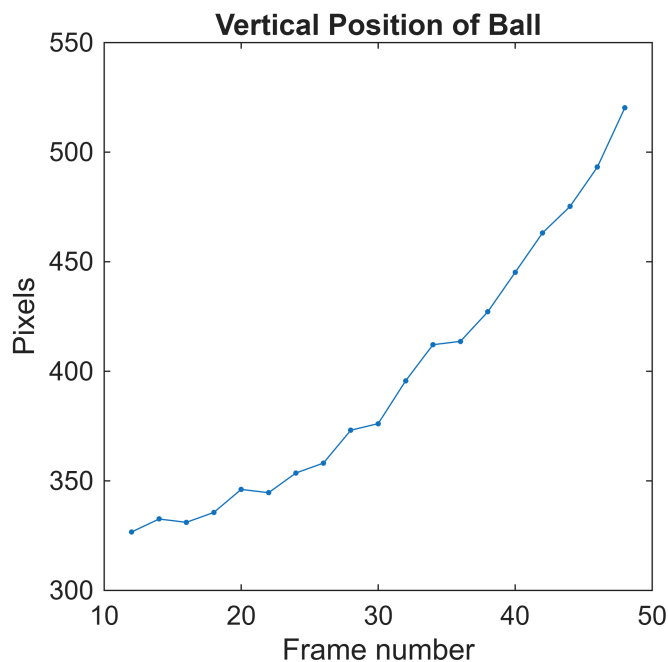
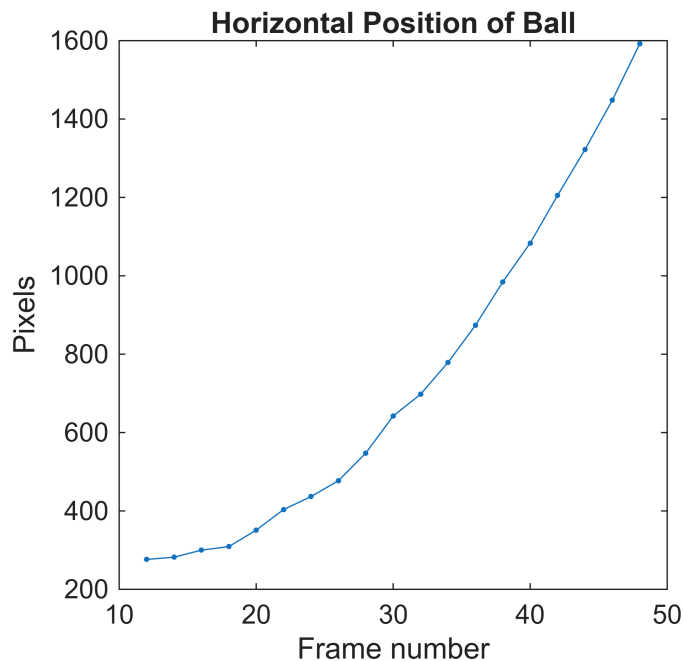
1. Measuring Motion along the Inclined Plane

We will be following in Galileo's footsteps, but instead of measuring our ball's motion with a water clock, we will take advantage of the frame rate of a video recording of the rolling ball. By observing the position of the ball in several frames of the video, we can analyze its motion along the path of the ramp.

Click the button to create a pop-up figure showing the motion of the ball rolling down the ramp, frame by frame. In each frame, click on the position of the ball. MATLAB will automatically advance to the next frame after each click.

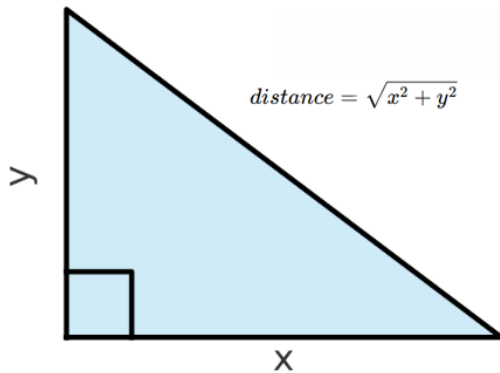
We will create our first plots of our data for this experiment. When plotting data, it's important to be aware of the units used to describe the data. Our observations are capturing the position of the ball in image pixels over several frames of the video, so the x-axis representing time is labeled 'Frame number' and the y-axis representing position is labeled 'Pixels.'

Click the button to plot the data.

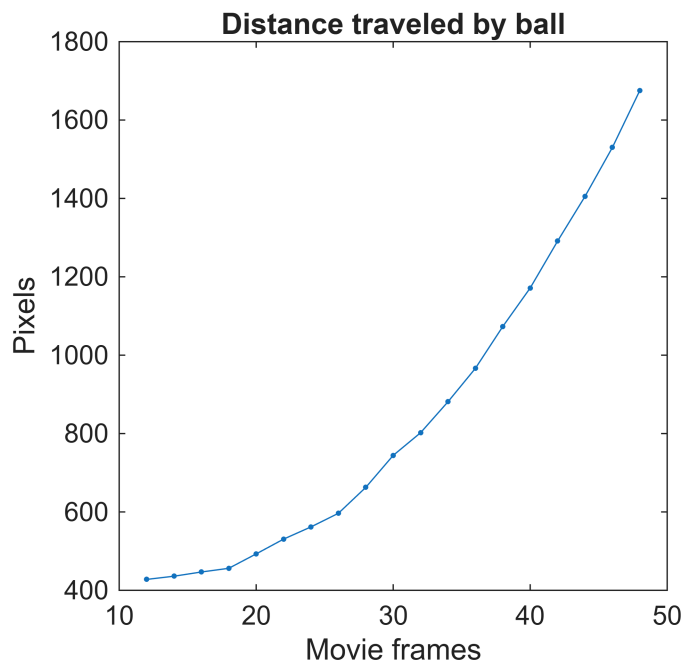


2. Calculate the distance traveled by the ball

We will use the Pythagorean theorem to calculate the distance traveled by the ball, not just the horizontal and vertical position.



Select the correct formula to apply to calculate distance from the menu:



3. Convert measurement units

Now we will convert from pixels and movie frames into more useful physical units. When we loaded the video, we also read the video's frame rate, the number of frames displayed per second. We can use this rate of 30 frames per second to convert the time data into seconds.

$$\text{Time in seconds} = \frac{\text{frames}}{\text{frame rate}} = \frac{\text{frames}}{30 \text{ frames per second}}$$

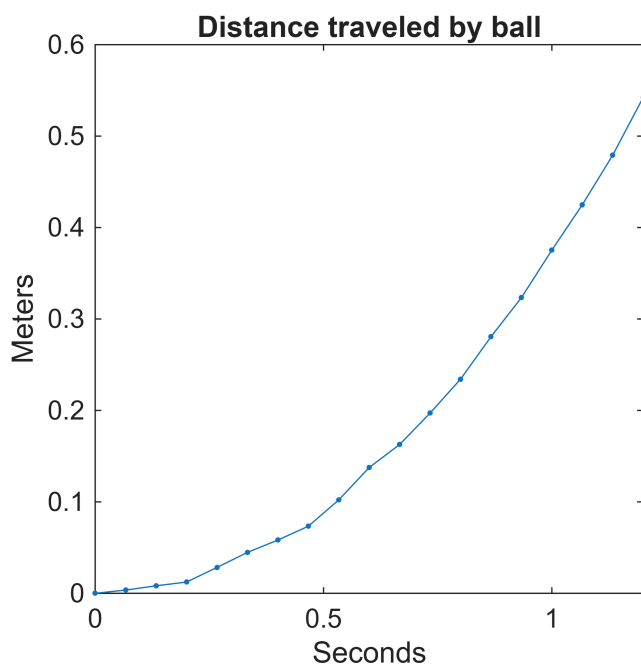
The pixels can be converted as well. One centimeter in the ruler shown in the video covers 23 pixels, so 2,300 pixels is one meter.

$$\text{Distance in meters} = \frac{\text{pixels}}{2300 \text{ pixels per meter}}$$



We will also subtract the first values for time and distance from the data so that we begin from $t = 0$ seconds and $d = 0$ meters.

Click the button to apply these unit conversions.



4. Fit Data with a Model

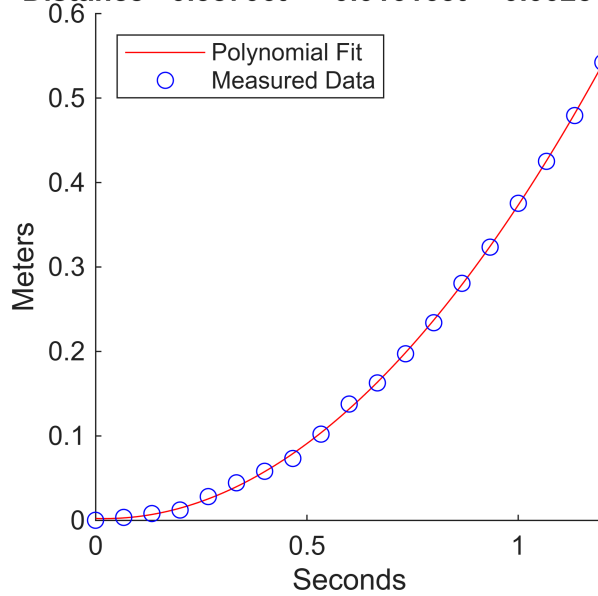
Next, we will fit our data with a mathematical model to produce an equation that represents the relationship between time and distance traveled. We will fit the data with a polynomial, but we must choose which degree polynomial best describes the data. Keep in mind the kinematics equation:

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

Select the degree of polynomial to use to fit the distance traveled over time, using the drop-down control:

Distance traveled by ball

$$\text{Distance} = 0.38706t^2 + -0.016163t^1 + 0.0023407 \text{ m}$$



5. Calculate velocity

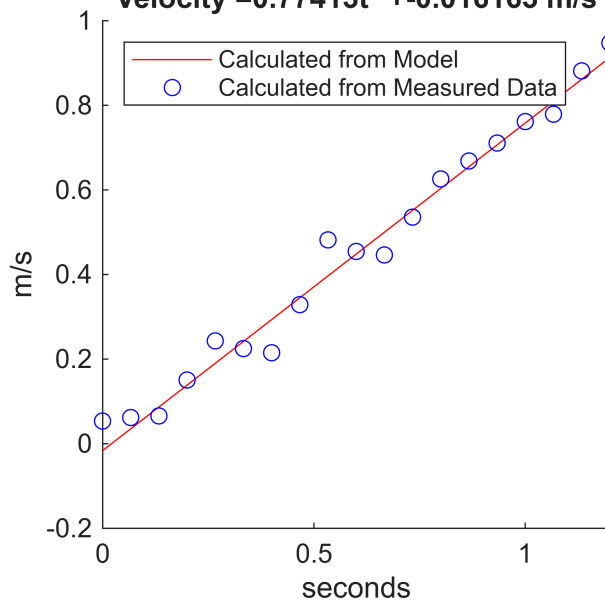
The velocity can be calculated by differentiating the polynomial fit with respect to time. We can also find the average velocity from the measured data points by finding the change in distance divided by the change in time in the measured data set:

$$\bar{v} = \frac{\Delta d}{\Delta t}.$$

Click the button to calculate and plot both the velocity obtained from the model and the average velocity calculated from the data.

Velocity of ball

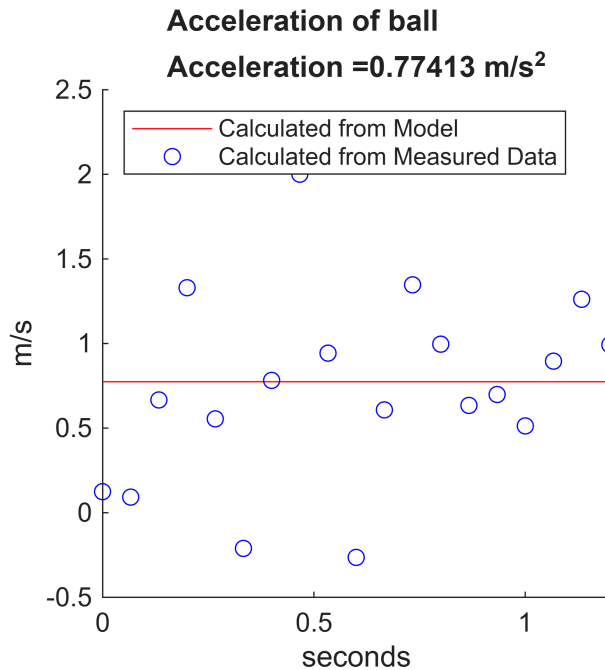
$$\text{Velocity} = 0.77413t^1 + -0.016163 \text{ m/s}$$



6. Calculate acceleration

We can follow a similar procedure to obtain acceleration, both by differentiating the equation for velocity with respect to time and by calculating the average acceleration by finding the change in velocity divided by the change in time:

$$\bar{a} = \frac{\Delta \bar{v}}{\Delta t}.$$



Gravitational Acceleration on the Moon

During the Apollo 15 mission in 1971, astronaut David Scott performed a simple but powerful demonstration of Galileo's principle of free fall. Standing on the Moon, he dropped a geologic hammer and a falcon feather at the same time from the same height. Because the Moon has almost no atmosphere to produce air resistance, both objects experienced only the force of lunar gravity and fell side by side, hitting the surface simultaneously. This striking visual proved that, in the absence of air drag, all objects accelerate at the same rate under gravity regardless of their mass or composition. The experiment vividly confirmed one of the most fundamental concepts in physics and offered a memorable real-world validation of Newton's and Galileo's laws of motion.

1. Observing the Falling Hammer and Feather

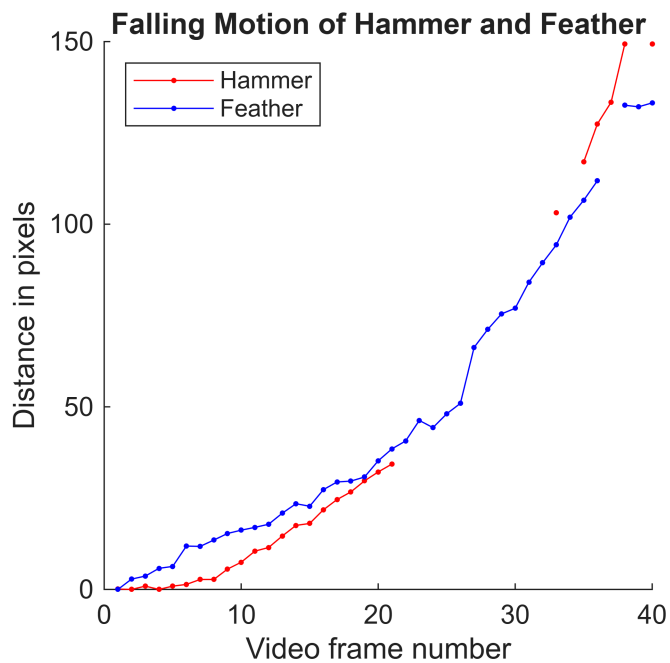
In the following frames from the archival Apollo video, David Scott can be seen simultaneously dropping a hammer and a feather. Because this is Apollo footage, the frames are low-resolution and it can be difficult or impossible to see either object in every frame. The approximate location of the hammer is marked with a red circle and the feather is indicated with a blue circle.

Video source: <https://moon.nasa.gov/resources/331/the-apollo-15-hammer-feather-drop/>

Frame 41 of 41



Plot the distance fallen of both the hammer and the feather with each frame of the video.



2. Unit Conversion

Now we will convert from the units associated with measuring values from the movie frames to more useful physical units. When we loaded the video, we also read the video's frame rate of 30 frames per second. We can use this to convert the time data into seconds.

$$\text{frame rate} = \frac{\text{frames}}{\text{second}}$$

We can modify this equation to convert the frames to seconds:

$$\text{Time in seconds} = \frac{\text{frames}}{\text{frame rate}}$$

so dividing our frame numbers by 30 will convert them into seconds.

We can also convert the distance measurements from pixels into meters. The *Apollo 15 Preliminary Science Report* (Allen, J. P. (1972). Summary of scientific results. In *Apollo 15 preliminary science report* (NASA SP-289, pp. 2-11). National Aeronautics and Space Administration) includes the following description of David Scott's experiment:

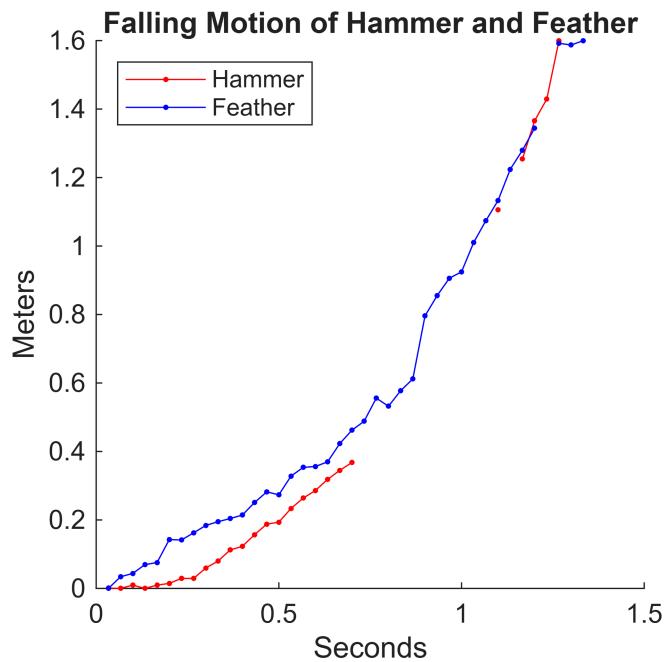
During the final minutes of the third extra-vehicular activity, a short demonstration experiment was conducted. A heavy object (a 1.32-kg aluminum geological hammer) and a light object (a 0.03-kg falcon feather) were released simultaneously from approximately the same height (approximately 1.6 m) and were allowed to fall to the surface. Within the accuracy of the simultaneous release, the objects were observed to undergo the same acceleration and strike the lunar surface simultaneously, which was a result predicted by well-established theory, but a result nonetheless reassuring considering both the number of viewers that witnessed the experiment and the fact that the homeward journey was based critically on the validity of the particular theory being tested.

From this description, we can approximate the total distance fallen as 1.6 meters, so our conversion factor is given by:

$$\text{Pixels per meter} = \frac{\text{Maximum pixel distance}}{1.6 \text{ m}}$$

$$\text{Distance in meters} = \frac{\text{pixels}}{\text{pixels per meter}}$$

Click the following button to apply these conversions:

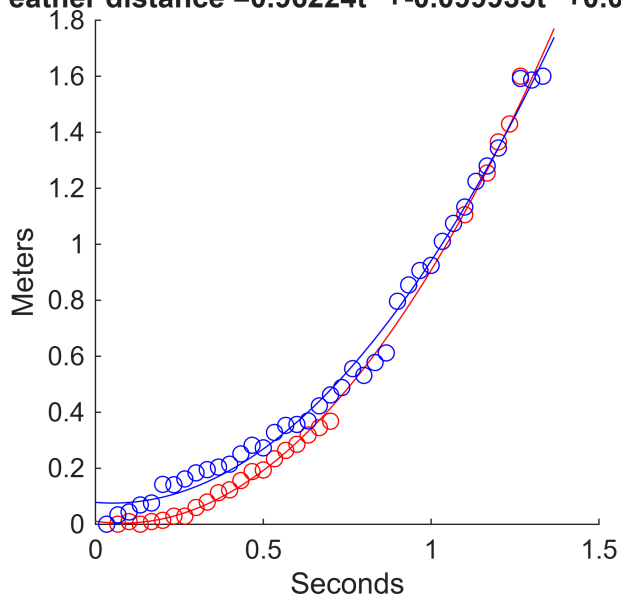


3. Fit Data with a Model

Next, we will fit our data with a mathematical model so that we have an equation that represents the relationship between time and the distance each object has fallen. We will fit the data with a polynomial representing the kinematic equation

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

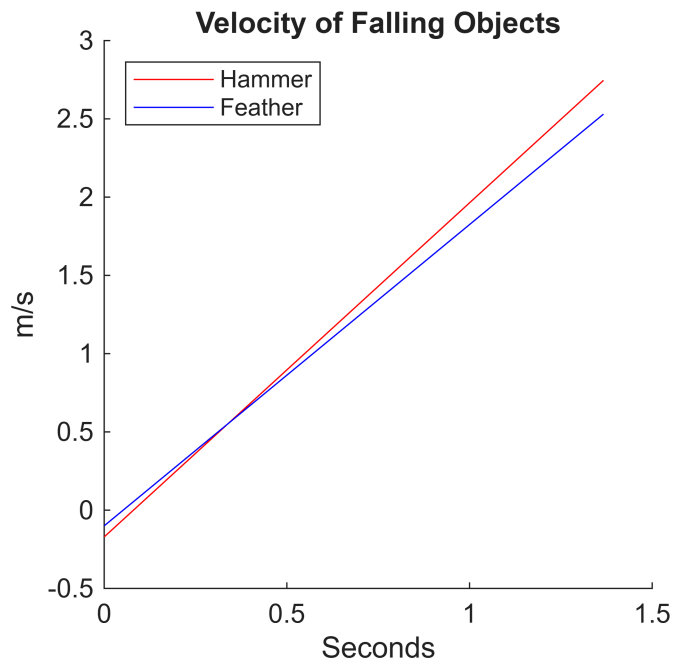
Hammer distance = $1.0671t^2 + -0.17087t^1 + 0.011099$
Feather distance = $0.96224t^2 + -0.099933t^1 + 0.078411$



4. Calculate Velocity

We can find the velocity of the hammer and the feather by taking the derivative with respect to time of the two functions for distance as a function of time:

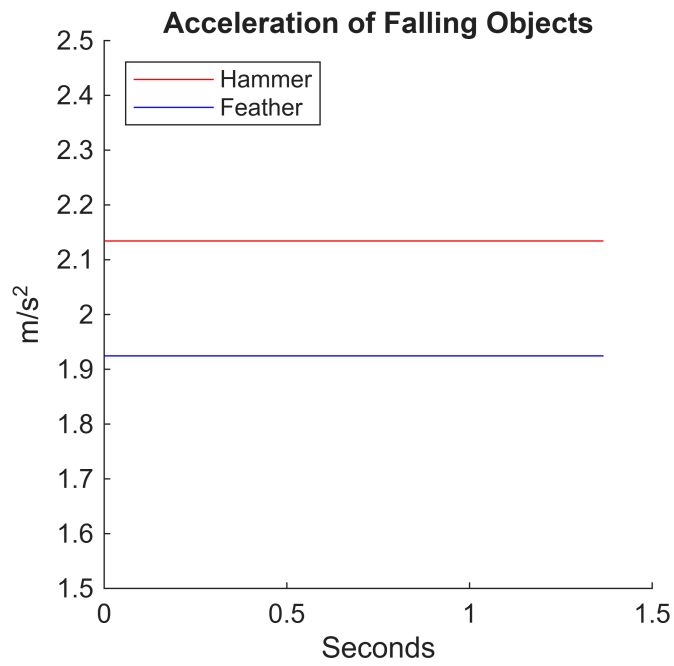
$$v = \frac{d}{dt} x(t)$$



5. Calculate Acceleration

The acceleration of the hammer and the feather can be found by taking the derivative with respect to time of the two functions for velocity with time:

$$a = \frac{d}{dt} v(t)$$



Gravitational acceleration of hammer: 2.13 meters/s²

Gravitational acceleration of feather: 1.92 meters/s²