Course: Mechanical Engineering Computation

Duration: ~1 class period (20–40 minutes)

Focus: MATLAB programming, first-order ODEs, thermal modeling

🟤 Objective

In this activity, you will simulate how coffee cools in different types of mugs using MATLAB. You will apply Newton's Law of Cooling to analyze how mug properties affect the rate of heat loss. The main goal is to practice implementing physical models using MATLAB's numerical solvers and plotting tools

📚 Background

Newton's Law of Cooling describes how the temperature of a hot object decreases over time as it exchanges heat with its environment. The rate of temperature change is proportional to the temperature difference between the object and the surrounding environment.

The governing equation is:

$$\frac{dT}{dt} = \frac{-1}{\tau}(T(t) - T_{room})$$

Where:

- $T(t)$: Coffee temperature at time t (in °C)
- T_{room}: Ambient temperature (assume 22 °C)
- τ: Time constant (in seconds), calculated as:

$$\tau = \frac{m}{h}.\frac{C_p}{A}$$

🧮 Provided Parameters

Parameter	Value	Unit	Notes
Initial coffee temperature T_0	85	°C	Hot fresh coffee
Ambient temperature T_{room}	22	°C	Room temperature
Mass of coffee m	0.35	kg	350 mL ≈ 1.5 cups
Specific heat C_p	4186	J/kg·°C	Approximated as water
Surface area A	0.03	m²	Top surface of coffee exposed
Heat transfer coefficient h	See below	W/m²·°C	Varies by mug

Figure 1: Mug Types and Corresponding h

🧪 What You Will Do

Step 1: Setup in MATLAB

Open MATLAB and create a script file (e.g., coffee_cooling_sim.m). You will define parameters, compute τ, and simulate the cooling process using ode45.

Step 2: Implement the ODE

Use the following MATLAB function structure:

```
% Parameters
T_initial = 85;
T_room = 22;
m = 0.35;
Cp = 4186;
A = 0.03;
h = 10;  % change this to try different mugs

% Time constant
tau = (m * Cp) / (h * A);

% ODE definition and solution
tspan = [0 1800]; % 30 minutes
T0 = T_initial;

[t, T] = ode45(@(t, T) -(1/tau)*(T - T_room), tspan, T0);
```

Step 3: Plot the Results

Plot the temperature curve vs. time in minutes:

```
plot(t/60, T, 'LineWidth', 2)
xlabel('Time (minutes)')
ylabel('Temperature (°C)')
title('Cooling of Coffee in a Mug')
grid on
```

Step 4: Repeat for Other Mug Types

Try all three values of h and **compare the results on a single plot**. Use the hold on command and add a legend.

✍️ What to Submit

1. Your **MATLAB script** (.m file).
2. A **plot** comparing cooling curves of at least **two mug types** (label and include legend).
3. Short written answers to the following:

📄 Questions

- Which mug kept the coffee warm the longest? Why?
- What happens to the time constant τ as h increases?
- How would adding a lid (reducing exposed area A) change the outcome?
- Suggest one way to improve the realism of this model.

🧠 Extension Ideas (Optional)

- Add a function to compute the time to reach 60°C (optimal drinking temp).
- Simulate the effect of room temperature fluctuation (e.g., air conditioning).
- Create a version using Euler's method instead of ode45.