Part 1: Modeling the Monarch Population Using Matrices

In this activity, we will explore modeling a monarch butterfly population by breaking the population into three life-stages and utilizing matrices. After we set-up our model, we will make use of MATLAB for calculations, automation, and for creating visualizations. Later in the semester, we will come back to this model and conduct further analysis (Part 2).

Learning Goals

- Construct, utilize, and interpret a matrix-based population model using concepts from linear algebra.
- Learn how to use MATLAB to generate model data, create visualizations, and draw conclusions.
- Explore how changes in survival rates affect long-term outcomes according to the model.
- Apply newly acquired knowledge of this modeling approach to an example of one's choosing.

Background

Once an egg hatches, a monarch butterfly goes through three life stages

Larva (caterpillar)

Pupa (chrysalis)

Adult (butterfly)

Monarchs spend (roughly) two weeks in each of the first two stages, and then have a lifespan of up to 6 weeks as an adult butterfly. In this activity, we will focus on building and exploring a model representing the population of monarchs over a period of time, broken into each of these three life stages. Utilizing data from previous studies on the life stages of the monarch, we make the following assumptions regarding the survival and reproduction rates

- 3.426% of larvae (caterpillars) survive to the pupa stage due to food shortages and predation.
- The survival rate of adult (butterfly) monarchs over a two-week span is 24.58%.
- 85% of pupae (chrysalises) will hatch into adults (butterflies).
- A female monarch butterfly will lay on average around 600 eggs in her lifetime (a span of 6 weeks). Only 45% of these eggs will be fertile and hatch into larvae.
- At any life-stage, there are roughly equal numbers of male and female monarchs.

Our goal is to utilize the data above to predict the number of monarchs in each life-stage over a period of time. Assume that the monarchs live in an indoor sanctuary (so we do not have to worry about modeling migration or changing seasons). Given the data, it makes sense to measure time in 2-week intervals. Define the following variables

- l_k = number of larvae (caterpillars) at $2k$ weeks
- p_k = number of pupae (chrysalises) at $2k$ weeks
- a_k = number of adults (butterflies) at $2k$ weeks.

The variables l_0, p_0, a_0 represent the initial populations in each life stage.

Activity

We will complete this activity in a MATLAB live script. Open the template `monarch_population.mlx` and save it in your class folder. Each problem has it's own section in this template.

1. Suppose an indoor, climate controlled, butterfly sanctuary starts with 1000 caterpillars ($l_0 = 1000, p_0 = 0, a_0 = 0$). Using the survival and reproduction data, estimate how many monarchs are in each life stage after 2 weeks (l_1, p_1, a_1), 4 weeks (l_2, p_2, a_2), and 6 weeks (l_3, p_3, a_3). Explain how you calculate each estimate. For some calculations, paying attention to units will be helpful.

2. Write down a system of linear equations relating the life stage populations at time level k to $(k+1)$ (two weeks later). There should be 3 equations, $l_{k+1} = \ldots$, $p_{k+1} = \ldots$, and $a_{k+1} = \ldots$.

3. Define the state vector, $\mathbf{x}_k = (l_k, p_k, a_k)$. Show that the system of linear equations you found can be represented as a matrix-vector equation in the form $A\mathbf{x}_k = \mathbf{x}_{k+1}$ for a particular matrix A.

4. Using MATLAB and the matrix-vector formulation, repeat your calculations from the first question. Let the MATLAB variables `x0`, `x1`, `x2`, and `x3` represent the populations at 0, 2, 4, and 6 weeks. How do these compare to your earlier calculations?

5. Examine the MATLAB code in this section. Unsuppress the output `P` and run the code. Examine the output and then add comments using `%` describing what each line of code is doing. Once you finish, suppress the output `P`, then write a 1-2 sentences summarizing what this code accomplishes.

6. Using the `plot` command, create a single graph containing the larva, chrysalis, and adult populations over the course of one year assuming that you start with 1000 larva. Then, add a plot of the total population to your graph (you might find the command `sum` helpful here). Add a title, axis labels, and a legend. What are the populations in each life stage after one year? What is the total population in the sanctuary after one year?

7. Suppose that the survival rate of larvae in the sanctuary is 1.5%.
   (a) Adjust your model accordingly and create a new plot. Determine the populations in each stage after one year. Is the sanctuary thriving?
   (b) What are some things that might be happening in the butterfly sanctuary that could lead to a survival rate that is lower than the originally suggested data?

8. Is there a critical larval survival rate for the long-term viability of the butterfly sanctuary? To explore this, define a slider variable, `L`, in your live script (go to Insert and select Control). Let this variable range from 0.015 to 0.04 in increments of 0.0001. Define `A(2,1) = L` and copy your plot code from problem 6. Using the slider, experimentally determine the value of L that corresponds to the long-term viability of the butterfly sanctuary. It may be helpful to extend your time period.

9. Utilize AI to research another species (animal or plant) that can be described in terms of 3 or 4 life-stages. Construct a matrix population model like we did for monarchs. You will need to decide on an appropriate time scale and choose some plausible survival/fertility rates.
   - Write your own short background description of your chosen species' life-cycle and survival/fertility rate statements, similar to the bullet points provided for the monarch population. Provide documentation for your model choices (links to direct sources are fine).
   - Define the variables in your state vector, $\mathbf{x}_k$. Explain how to get from your chosen survival/reproductive rate statements to a linear system of equations, and then to your matrix-vector model, $A\mathbf{x}_k = \mathbf{x}_{k+1}$.
   - Create a plot of your population (broken into life-stages) over an appropriate length of time using your choice of initial population data. Provide a long-term prediction for your model.