

Experimental Mathematics

Collatz Conjecture

GW#3

In this Group Work, we will explore the behavior of an integer sequence, also known as a *hailstone sequence*, through the computer experiment. The hailstone sequence is defined by the following simple rules:

1. Start with a positive integer (the *seed*).
2. If the current number is *even*, divide it by 2.
3. If the current number is *odd*, multiply it by 3 and add 1.
4. Repeat the process. (If the sequence reaches to 1, then stop the process.)

For example, starting with the seed, $n = 12$, one gets the sequence 12, 6, 3, 10, 5, 16, 8, 4, 2, 1.

Surprisingly, it seems that the sequence always reaches to 1, in finite steps, for any positive integer seed! (Which is known as the *Collatz Conjecture*.) This Collatz Conjecture has not been proven and remains one of mathematics' most famous unsolved problems.

1. **Generate the Matlab Code:** Use the Generative AI, such as ChatGPT, Copilot, and Gemini, etc., generate a Matlab code which can produce the hailstone sequence for any given positive sequence. Each person choose a different AI tool to generate the Matlab code. Can you explain the code to your group members line by line? (Often, even the same AI tool, it may generate a slightly different code.) What are the difference between codes by the different AI tools? Through the group discussion, choose the best code for experiment.
2. **Verify the Matlab code:** Try to check whether the code works by testing a few seed numbers of your choice. That is, you choose a seed number and generate the sequence by hand. The check whether it agrees with the computer generated sequence.
3. **Experiment the Matlab code:** By adding a loop control over the code, experiment the Collatz Conjecture from 1 to 1 million (10^6).
4. **More Experiment:** You noticed that the length of each sequence is different. Can you find any patterns of the sequence (or the length of the sequence) from the above experiment? Try to visualize the results (For instance, use the plot command that we introduced last week.) to observe the pattern.
5. **Make Your Own Sequence Rule:** Modify the rules to generate the hailstone sequence. For example, use mod 3, instead of mod 2, and define different way to generate the next number in the sequence, so that the resulting sequence is not so trivial. Be creative!
6. **Find Patterns or Conjectures:** Modify the Matlab code to adopt your new rules. Experiment with the computer to find new patterns or conjectures.