MATH 448 SCIENTIFIC COMPUTING
MATLAB ASSIGNMENT

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Learning Goals. The goal of this assignment is for students to demonstrate understanding of numerical interpolation through an intensive writing prompt. Two methods are implemented in MATLAB, Lagrange interpolation, and Neville's method. Students will critique the `.m`files and interpret the results in the context of traffic density data interpolation.

Context for Use. This assignment is offered as homework in conjunction with skills-practice homework from classical numerical analysis modules on numerical interpolation. Students in this course are typically math and/or computer science majors in their junior or senior year of undergraduate studies. Providing an appendix is *optional* and only necessary if students do not have the access to run the program themselves.

Materials. Students prepare for this assignment by doing typical numerical interpolation homework, including developing their own `.mfiles`that perform numerical interpolation. The assignment instructions are included below. Other material to provide includes the data file `uTrafficData.xlsx`. Alternatively, this can be done as an in-class assignment in small groups.

Assessment. Mathematics Writing Rubric (can be provided) is used to grade assignment. The aim is for students to demonstrate their understanding of interpolation (pros and cons), the benefits of different methods, and to do so in writing. Rubric available upon request.

Instructions. Create a technical written report to answer the following prompt using complete sentences. Submit this assignment on the online classroom management system (OCMS). Use the LaTeX HW template to complete this assignment. Submit only one cohesive PDF document.

Introduction. In this assignment you are asked to analyze the interpolation results used to obtain an estimate of traffic conditions within a two hour time span along Highway 101 in California. For simplicity, we are considering traffic conditions at a given location and *time* is represented discretely by the nodes T_i for $i = 1, \dots 21$, and evenly spaced between the interval from $[1, 3]$. Hence, the difference between two time nodes is $\frac{1}{10}$ hours. The corresponding values collected in a vector $\vec{Y}$ correspond to values measuring traffic density and are stored in the file found on our OCMS `uTrafficData.xlsx`.The values can range from -1 to 1. Values closer to 1 indicate less traffic. Values closer to -1 indicate heavy traffic.

Instructions. Your goal is to *summarize* the results obtained using two methods, each attempting to produce the Lagrange interpolating polynomial evaluated at time $T = 1.55$ in technical writing style. Compare the two methods and identify traits that make the method more or less desirable than the other. What do the results indicate about traffic conditions at this specific time? Which method do you recommend? Which result do you anticipate is the most accurate and why? Which implementation is the best and why? (2 page max)

The script below is named `Lagrange_uTrafficData.m`.It calculates $P(1.55)$ by Lagrange interpolation. Below it is the MATLAB command window result from two different runs.

```
tic
format short
filename = 'uTrafficData.xlsx';
sheet = 2;
%T = [1:1/10:3]; %Time increments equally spaced between 1 and 3. These are the nodes t_k.
%A = 'A3:A23'; % Traffic data at one location. Values are in the range between -1 and 1. ↩
    Values closer to 1 indicate less traffic.
T = [1:1/10:2];
A = 'A3:A13';
Y = xlsread(filename,sheet,A);
MaxU = max(Y)
MinU = min(Y)
N =length(T);
t =1.55;
TK=zeros(N,N-1);
for k=1:N
    TK(k,:)=T([1:k-1 k+1:end]); %removes the k'th node
end
TK;
NUMk=t-TK; %we will need t - t_i where we have removed the kth node.
NUM=prod(NUMk,2); % This produces the product which becomes our numerator for each L_k
DENk=zeros(N,N-1);
for i=1:N
    DENk(i,:)=T(i)-TK(i,:);
end
DEN=prod(DENk,2); %The product which makes up the denominator of each L_k organized in a ↩
    vector
L=zeros(N,1);
for i=1:N
    L(i)=NUM(i)/DEN(i);
end
L;
Q=L.*Y; %elementwise multiplication to form each term in the summand that becomes the nth ↩
    interpolating polynomial evaluated at t.
P=sum(Q) %The answer
toc
```

Below is the first run of `Lagrange_uTrafficData.m`.In total, 21 time nodes were used.

```
>> Lagrange_uTrafficData
MaxU =
    0.8080

MinU =
    0.6116

P =
    0.7802
```

```
Elapsed time is 0.068787 seconds.
>>
```

This is the second run. In this run, 11 time nodes were used. These corresponded to T_i for $i = 1, \ldots, 11$.

```
>> Lagrange_uTrafficData
MaxU =
    0.8080

MinU =
    0.6736

P =
    0.8014
Elapsed time is 0.056448 seconds.
>>
```

Finally, Neville's method was applied. Neville's method was also implemented twice. The first run used all 21 nodes and the second run used the first 11 nodes.

```
function[Qnn]=Nev(t,T,Y) %applying Neville's Method to find P(t) for the given nodes in T ↩
    and the corresponding values in Y.
tic
format short
N=length(T);
Q0=zeros(1,N); %N=n+1
X=zeros(1,N+1);
for i=1:N
    X(i)=T(i);
end
for i=1:N
    Q0(i)=Y(i);
end
Q=zeros(N+1,N+1);
for i=1:N
    Q(i,1)=Q0(i);
end

D = zeros(1,N+1);
D(1) = t-X(1);

 for i = 1:N
    D(i+1) = t-X(i+1);
    for j = 1:i
        Q(i+1,j+1) = ((D(i+1)*Q(i,j)-D(i+1-j)*Q(i+1,j))/(D(i+1)-D(i-j+1)));
    end
 end
 Qnn=Q(N,N);
 toc
return

%Practice example from Class
%X=[8.1,8.3,8.6,8.7]
%F=[16.94410, 17.56492, 18.50515, 18.82091]
```

```
>> Nev(1.55,T,Y)
```

```
Elapsed time is 0.000028 seconds.
ans =
    0.7802
>>
>> Nev(1.55,T,Y)
Elapsed time is 0.000022 seconds.
ans =
    0.8014
```

Appendix. For reference, the following command window results display information on how `Lagrange_uTrafficData.m` is running. Below is the case where 11 nodes are considered. You are encouraged to run the program on your computer.

```
>> Lagrange_uTrafficData

Y =

    0.7774
    0.8006
    0.8076
    0.7680
    0.7848
    0.8080
    0.7910
    0.7922
    0.7792
    0.7586
    0.6736


MaxU =

    0.8080


MinU =

    0.6736


TK =

  Columns 1 through 9

    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000    1.8000    1.9000
    1.0000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000    1.8000    1.9000
    1.0000    1.1000    1.3000    1.4000    1.5000    1.6000    1.7000    1.8000    1.9000
    1.0000    1.1000    1.2000    1.4000    1.5000    1.6000    1.7000    1.8000    1.9000
    1.0000    1.1000    1.2000    1.3000    1.5000    1.6000    1.7000    1.8000    1.9000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.6000    1.7000    1.8000    1.9000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.7000    1.8000    1.9000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.8000    1.9000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000    1.9000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000    1.8000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000    1.8000

  Column 10

    2.0000
```

```
    2.0000
    2.0000
    2.0000
    2.0000
    2.0000
    2.0000
    2.0000
    2.0000
    2.0000
    1.9000


NUMk =

  Columns 1 through 9

    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500   -0.2500   -0.3500
    0.5500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500   -0.2500   -0.3500
    0.5500    0.4500    0.2500    0.1500    0.0500   -0.0500   -0.1500   -0.2500   -0.3500
    0.5500    0.4500    0.3500    0.1500    0.0500   -0.0500   -0.1500   -0.2500   -0.3500
    0.5500    0.4500    0.3500    0.2500    0.0500   -0.0500   -0.1500   -0.2500   -0.3500
    0.5500    0.4500    0.3500    0.2500    0.1500   -0.0500   -0.1500   -0.2500   -0.3500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.1500   -0.2500   -0.3500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.2500   -0.3500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500   -0.3500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500   -0.2500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500   -0.2500

  Column 10

   -0.4500
   -0.4500
   -0.4500
   -0.4500
   -0.4500
   -0.4500
   -0.4500
   -0.4500
   -0.4500
   -0.3500


NUM =

   1.0e-06 *

   -0.0872
   -0.1066
   -0.1370
```

```
   -0.1919
   -0.3198
   -0.9593
    0.9593
    0.3198
    0.1919
    0.1370
    0.1066


DEN =

   1.0e-03 *

    0.3629
   -0.0363
    0.0081
   -0.0030
    0.0017
   -0.0014
    0.0017
   -0.0030
    0.0081
   -0.0363
    0.3629


L =

   -0.0002
    0.0029
   -0.0170
    0.0634
   -0.1851
    0.6662
    0.5552
   -0.1057
    0.0238
   -0.0038
    0.0003


P =

    0.8014

>> Lagrange_uTrafficData

Y =
```

```
    0.7774
    0.8006
    0.8076
    0.7680
    0.7848
    0.8080
    0.7910
    0.7922
    0.7792
    0.7586
    0.6736
    0.6512
    0.6336
    0.6122
    0.6232
    0.6192
    0.6462
    0.6156
    0.6208
    0.6116
    0.6132


MaxU =

    0.8080


MinU =

    0.6116


TK =

  Columns 1 through 8

    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000    1.8000
    1.0000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000    1.8000
    1.0000    1.1000    1.3000    1.4000    1.5000    1.6000    1.7000    1.8000
    1.0000    1.1000    1.2000    1.4000    1.5000    1.6000    1.7000    1.8000
    1.0000    1.1000    1.2000    1.3000    1.5000    1.6000    1.7000    1.8000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.6000    1.7000    1.8000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.7000    1.8000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.8000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000
```

```
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000
    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000    1.7000

Columns 9 through 16

    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000    2.6000
    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000    2.6000
    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000    2.6000
    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000    2.6000
    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000    2.6000
    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000    2.6000
    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000    2.6000
    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000    2.6000
    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000    2.6000
    1.8000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000    2.6000
    1.8000    1.9000    2.1000    2.2000    2.3000    2.4000    2.5000    2.6000
    1.8000    1.9000    2.0000    2.2000    2.3000    2.4000    2.5000    2.6000
    1.8000    1.9000    2.0000    2.1000    2.3000    2.4000    2.5000    2.6000
    1.8000    1.9000    2.0000    2.1000    2.2000    2.4000    2.5000    2.6000
    1.8000    1.9000    2.0000    2.1000    2.2000    2.3000    2.5000    2.6000
    1.8000    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.6000
    1.8000    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000
    1.8000    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000
    1.8000    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000
    1.8000    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000
    1.8000    1.9000    2.0000    2.1000    2.2000    2.3000    2.4000    2.5000

Columns 17 through 20

    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
```

```
    2.7000    2.8000    2.9000    3.0000
    2.7000    2.8000    2.9000    3.0000
    2.6000    2.8000    2.9000    3.0000
    2.6000    2.7000    2.9000    3.0000
    2.6000    2.7000    2.8000    3.0000
    2.6000    2.7000    2.8000    2.9000


NUMk =

  Columns 1 through 8

    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500   -0.2500
    0.5500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500   -0.2500
    0.5500    0.4500    0.2500    0.1500    0.0500   -0.0500   -0.1500   -0.2500
    0.5500    0.4500    0.3500    0.1500    0.0500   -0.0500   -0.1500   -0.2500
    0.5500    0.4500    0.3500    0.2500    0.0500   -0.0500   -0.1500   -0.2500
    0.5500    0.4500    0.3500    0.2500    0.1500   -0.0500   -0.1500   -0.2500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.1500   -0.2500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.2500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500
    0.5500    0.4500    0.3500    0.2500    0.1500    0.0500   -0.0500   -0.1500

  Columns 9 through 16

   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500   -1.0500
   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500   -1.0500
   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500   -1.0500
   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500   -1.0500
   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500   -1.0500
   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500   -1.0500
   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500   -1.0500
   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500   -1.0500
   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500   -1.0500
   -0.2500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500   -1.0500
   -0.2500   -0.3500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500   -1.0500
   -0.2500   -0.3500   -0.4500   -0.6500   -0.7500   -0.8500   -0.9500   -1.0500
   -0.2500   -0.3500   -0.4500   -0.5500   -0.7500   -0.8500   -0.9500   -1.0500
   -0.2500   -0.3500   -0.4500   -0.5500   -0.6500   -0.8500   -0.9500   -1.0500
```

```
  -0.2500   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.9500   -1.0500
  -0.2500   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -1.0500
  -0.2500   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500
  -0.2500   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500
  -0.2500   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500
  -0.2500   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500
  -0.2500   -0.3500   -0.4500   -0.5500   -0.6500   -0.7500   -0.8500   -0.9500

  Columns 17 through 20

  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.1500   -1.2500   -1.3500   -1.4500
  -1.0500   -1.2500   -1.3500   -1.4500
  -1.0500   -1.1500   -1.3500   -1.4500
  -1.0500   -1.1500   -1.2500   -1.4500
  -1.0500   -1.1500   -1.2500   -1.3500


NUM =

  1.0e-06 *

  -0.0558
  -0.0682
  -0.0877
  -0.1227
  -0.2046
  -0.6137
   0.6137
   0.2046
   0.1227
   0.0877
   0.0682
   0.0558
   0.0472
```

```
    0.0409
    0.0361
    0.0323
    0.0292
    0.0267
    0.0245
    0.0227
    0.0212


DEN =

    0.0243
   -0.0012
    0.0001
   -0.0000
    0.0000
   -0.0000
    0.0000
   -0.0000
    0.0000
   -0.0000
    0.0000
   -0.0000
    0.0000
   -0.0000
    0.0000
   -0.0000
    0.0000
   -0.0000
    0.0001
   -0.0012
    0.0243


L =

   -0.0000
    0.0001
   -0.0007
    0.0058
   -0.0407
    0.3911
    0.9777
   -0.6518
    0.6355
   -0.6052
    0.5178
   -0.3851
    0.2444
```

```
   -0.1304
    0.0575
   -0.0206
    0.0058
   -0.0013
    0.0002
   -0.0000
    0.0000


P =

    0.7802

>>
```