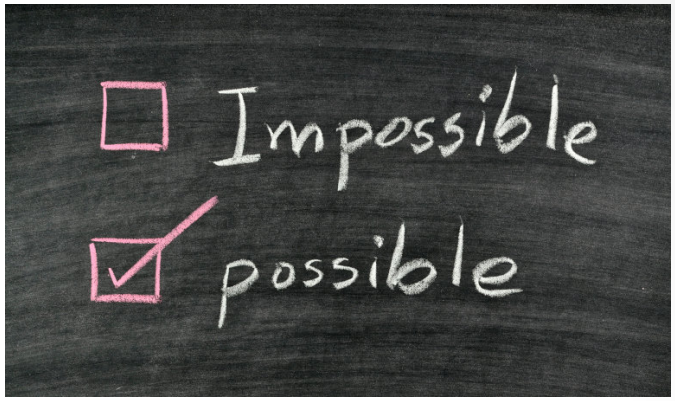

Doing the impossible? Generating productive discourse in college-level mathematics classrooms

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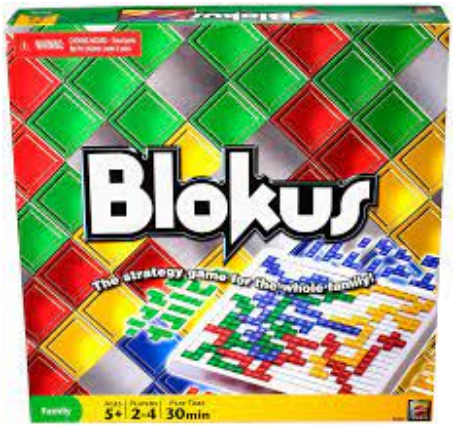
Today's goals

- Provide a vision of current recommendations for mathematics education
- Explore the idea of productive mathematics classroom discourse, including examples
- Generate and share ideas for how to facilitate productive mathematics discourse in college-level mathematics classrooms

A good mathematical discussion is like…



Game of your choice

Introductions

What game do you think represents a good mathematical discussion?

<u>In Breakout Rooms</u>

- Name/Position/Institution
- What brings you here this afternoon / What is your connection to this work?
- Explain why your game has one or more qualities of a good mathematics discussion
- What is a favorite classroom strategy of yours to encourage mathematical discussion?

"Correct answers matter, but not as indicators of who is able to do mathematics. Engaging in mathematical discourse is essential for developing mathematical identity and should be recognized as a better indicator of mathematical competence."

Robert Berry, President NCTM

Major shifts in Common Core State Standards for Mathematics (CCSS-M)

- Focus: Structure
- Coherence: Progressions
- Clarity: Fewer, more rigorous

- Includes standards for mathematical Content & Practices

The Standards for Mathematical Practice

Mathematically proficient students…

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practices

What do you notice about these standards?

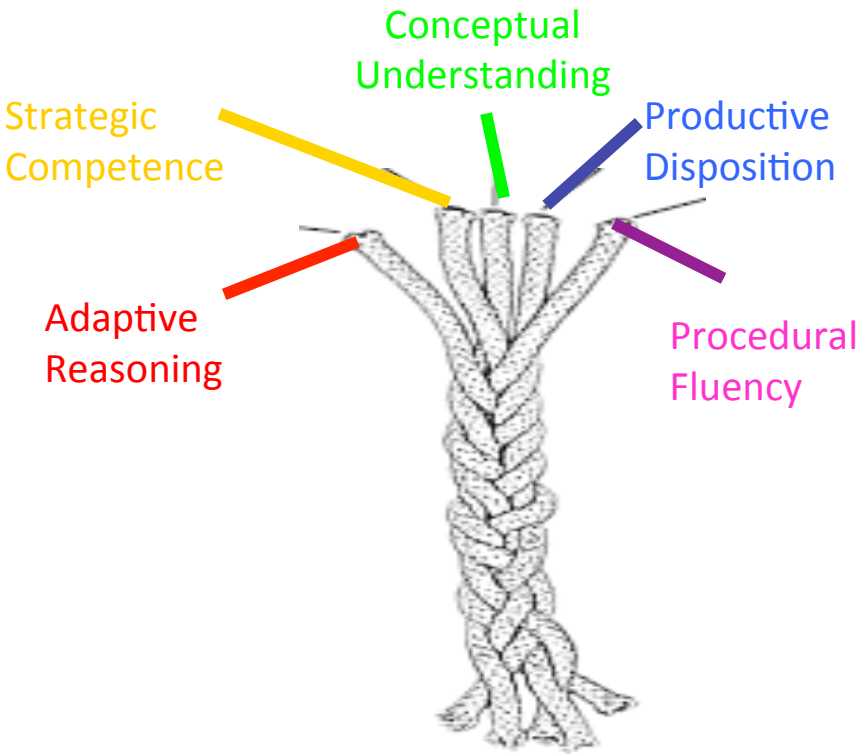
What questions do you have about them?

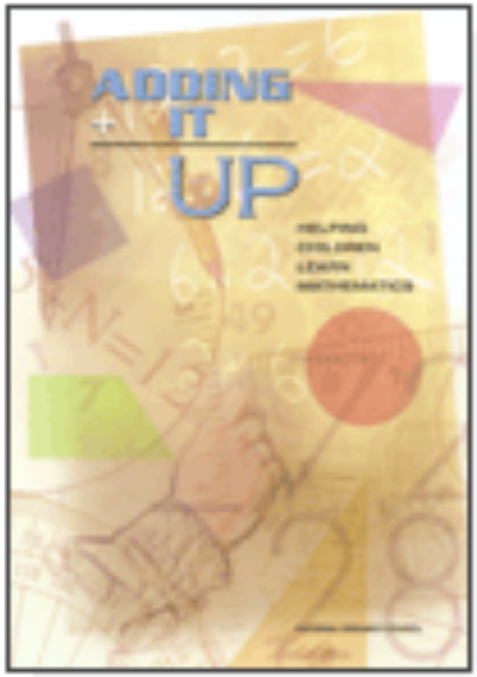
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The eight Standards for Mathematical Practice place an emphasis on students *doing* mathematics and *demonstrating* learning.

Underlying Framework

Strands of Mathematical Proficiency





Eight Effective Instructional Practices

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- Use and connect mathematical representations
- Facilitate meaningful mathematical discourse
- Pose purposeful questions
- Build procedural fluency from conceptual understanding
- Support productive struggle in learning mathematics
- Elicit and use evidence of student thinking

Eight Effective Instructional Practices

- Establish mathematics goals to focus learning
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What does it look like and sound like

- Class on Proof
- Goal is to determine the truth of a statement given knowledge of four cards that provide hints about its truth

http://collegemathvideocases.org/cases/case.php?VCID=8

Reflecting on the clip…

- What do you notice about this clip?
- What did the teacher do to engage students in mathematical practices?
- To what extent does the teacher:
  - Facilitate meaningful discourse
  - Pose purposeful questions
  - Promote productive struggle

Use of

- Repeating
- Paraphrasing
- Probing

Instructor sets up the task and provides some insights, but the majority of the discussion is based on student ideas

What does it look like and sound like in virtual (Zoom) setting

The setting

- Introduction to Mathematical Reasoning (Proofs)
- Flipped Instructional Model
  - Online lecture videos, Pre-class assessment, Active work in class
- Students are familiar with general proof techniques. Application to show one set is a subset of another.
- Beginning of (synchronous) class activity.

Why Flip?

- Flipping is less about watching videos and more about what you make time for in “face-to-face” class.
  - Better understanding of student thinking, more opportunities for students to justify their thinking.
  - Creates more space for peer learning and helping students when they’re deep in a question.
  - Students who learn from lectures have access to them when they are ready.

Adjustments for remote flipping

- There is no eavesdropping in Breakout Rooms.
  - Debriefs and paraphrasing with the larger group is even more important.
- Student sharing is a challenge.
  - Communicating math can be difficult without the right equipment.
  - Be open to students finding their preferred way to share with you.
- It can be easier for some students to hide in class.

Roll Film!

Reflecting on the clip…

- What do you notice about this clip?
- What did the teacher do to engage students in mathematical practices?
- What are some similarities in the two clips?
- To what extent does the teacher:
  - Facilitate meaningful discourse
  - Pose purposeful questions
  - Promote productive struggle

Break – 10 min.

After the break we will explore specific instructional tools and strategies

What do you Notice

What do you Wonder

What do you Notice

What do you Wonder

$$\frac{2}{x^2} + \frac{1}{x}$$

$$\frac{4}{x^2} - \frac{1}{x}$$

$\tan\Theta + \cot\Theta = \sec\Theta\csc\Theta$

Question variety

Which type(s) of questions do you ask most often?

- Inviting Participation
- Surfacing Student Ideas
- Probing Thinking
- Eliciting Answers / Assessing
- Gathering Information
- Encouraging Reflection and Justification

TALK MOVES – Specific actions taken by teachers to encourage talk

TABLE 3.2 PRODUCTIVE TALK MOVES FOR SUPPORTING CLASSROOM DISCUSSIONS

Talk Moves	What It Means and Why	Example Teacher Prompts
1. Revoicing	This move involves restating the statement as a question in order to clarify, apply appropriate language, and involve more students. It is an important strategy to reinforce language and enhance comprehension for ELLs.	"You used the hundreds chart and counted on?" "So, first you recorded your measurements in a table?"
2. Rephrasing	Asking students to restate someone else's ideas in their own words will ensure that ideas are stated in a variety of ways and encourage students to listen to each other.	"Who can share what Ricardo just said, but using your own words?"
3. Reasoning	Rather than restate, as in talk move 2, this move asks the student what they think of the idea proposed by another student.	"Do you agree or disagree with Johanna? Why?"
4. Elaborating	This is a request for students to challenge, add on, elaborate, or give an example. It is intended to get more participation from students, deepen student understanding, and provide extensions.	"Can you give an example?" "Do you see a connection between Julio's idea and Rhonda's idea?" "What if . . . "
5. Waiting	Ironically, one "talk move" is to not talk. Quiet time should not feel uncomfortable, but should feel like thinking time. If it gets awkward, ask students to pair-share and then try again.	"This question is important. Let's take some time to think about it."

Source: Based on Chapin, S., O'Conner, C., & Anderson, N. (2009). *Classroom Discussions: Using Math Talk to Help Students Learn* (2nd ed.). Sausalito, CA: Math Solutions. Reprinted with permission.

Question Types That Lead to Productive Discourse

Question type		Description	Examples
3	Making the mathematics visible	Students discuss mathematical structures and make connections among mathematical ideas and relationships.	What does your equation have to do with the band concert situation? How does that array relate to multiplication and division? In what ways might the normal distribution apply to this situation?
4	Encouraging reflection and justification	Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.	How might you prove that 51 is the solution? How do you know that the sum of two odd numbers will always be even? Why does plan A in the Smartphone Plans task start out cheaper but become more expensive in the long run?

From NCTM *Principles to Actions*

Question Types That Lead to Productive Discourse

Question type		Description	Examples
1	Gathering information	Students recall facts, definitions, or procedures.	When you write an equation, what does the equal sign tell you? What is the formula for finding the area of a rectangle? What does the interquartile range indicate for a set of data?
2	Probing thinking	Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.	As you drew that number line, what decisions did you make so that you could represent 7 fourths on it? Can you show and explain more about how you used a table to find the answer to the Smartphone Plans task? It is still not clear how you figured out that 20 was the scale factor, so can you explain it another way?

From NCTM *Principles to Actions*

Facilitate meaningful mathematical discourse.

Effective teaching of mathematics facilitates discourse among students in order to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Facilitate meaningful mathematical discourse Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse. Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments. Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. Identifying how different approaches to solving a task are the same and how they are different.

Support Productive Struggle in Learning Mathematics

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Support productive struggle in learning mathematics
Teacher and student actions

What are *teachers* doing?	What are *students* doing?
Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle. Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. Helping one another without telling their classmates what the answer is or how to solve the problem.

Pose purposeful questions

Effective teaching of mathematics uses purposeful questions to assess and advance student reasoning and sense making about important mathematical ideas and relationships.

Pose purposeful questions
Teacher and student actions

What are *teachers* doing?	What are *students* doing?
Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. Allowing sufficient wait time so that more students can formulate and offer responses.	Expecting to be asked to explain, clarify, and elaborate on their thinking. Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly. Reflecting on and justifying their reasoning, not simply providing answers. Listening to, commenting on, and questioning the contributions of their classmates.

Reflect

2 minute pause

How do these teaching practices support mathematical understanding?

- To what degree do you make use of these practices in your own instruction? Or have you seen them being used?
- Which practices do you find most helpful and why?

AND/OR

- Which do you think might be most challenging to implement?
- Which might you strive toward and why?

Productive Discourse – why is it important?

Productive discourse is used:

- To elicit and make student thinking visible
- To thoroughly explore and extend student thinking through the use of teacher questions
- To move students toward justifying mathematical reasoning, warranting interpretive claims, and constructing evidence-based explanations
- To help students make sense of the activity in which they are engaged

How do we get started … or do more?

- Make time for discussion
- Practice being quiet
- Don’t be afraid to fail
- Encourage rough draft talking
  - Ask students to share initial ideas, value mistakes & incomplete thinking as way to move learning forward
- Shift some content to out-of-class time
  - Identify the 2-3 lecture-intense topics you can offload to outside of class videos
- Implement specific routines
  - Turn and Talk
  - Notice and Wonder
  - Math Wellness Checks

Setting up classroom environment

Engaging students in meaningful discussion is hard work!!

It requires a substantial shift in typical modes of instruction

This is true not just for us as teachers, but also for students

Engagement and student input should eventually be expected, but initially needs to be worked on

- Shift Norms of participation
- Focus on what students are thinking/understanding rather than what you are teaching

Breakout rooms

Think about the strategies your group shared in the first breakout session.

- What new adaptations might you make?
- What might you start doing more of?
- What might you stop doing to make room for something new?

Plan for Action

- If you are already doing some of this - how might you improve or what might you add?

- If you are not already doing some of this, what first steps might you be able to do? (e.g., let students talk more at the beginning of class; ask questions that clarify student thinking)
- What "quick win" changes could you make to encourage more student discourse? What might you do with a longer term goal?

- What might you have to give up as a result of this change in your practice (e.g., the display of my own math expertise)