Teaching Quantitative Skills in a Geoscience Context
"Darcy's Law for multiple levels in Math and Geoscience Courses"
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Darcy's Law is a fundamental ground water flow equation, developed experimentally by Henri Darcy in 1856. In most introductory geology texts, Darcy's Law is usually presented as an equation with only vague explanation or justification. It is usually illustrated by simple figures showing a sloping water table, and ignoring the wide variability in values for hydraulic conductivity that exist in natural systems. The beauty of Darcy's Law at this level is that it can be used to illustrate a number of quantitative concepts without any actual numerical solution. As student sophistication increases, Darcy's Law can be revisited in a more quantitative context. In senior-level or introductory graduate level hydrogeology texts, Darcy's Law is presented as a second-order partial differential equation of a type that is often not covered until the third semester of most calculus sequences or in a differential equations course.

In this project, we present a series of modules that cover the concept of Darcy's Law at increasingly complex levels. The primary goal of this project is to develop Darcy's Law at different levels as students progress through their Geoscience and/or Mathematics curriculum.

Part IA: Introductory Geoscience (GEOS 120 – Dynamic Earth)
Part IB: Math for Earth & Life Science I (Math 150)

Concepts for Geoscience Class	**Concepts for Mathematics class**
Gradient (slope of the water table)	Slope of a line
Proportionality	Proportionality
Graphical relationships	Graphical relationships
Variation of parameters	Variation of parameters
Physical relationship of equation parameters	Scientific Notation & orders of magnitude
Units	Units

Note that we divide this part into a math section and a geoscience section at this level primarily because of the student audience: the math class is guaranteed to have science students enrolled, while the geoscience course has a relatively small number of science students when compared to the numbers of general-education students. In the math course, students will be expected to work with the equation to develop more quantitative skills. In the Geoscience course, we seek to help students become conceptually comfortable with mathematical expressions as they relate to the physical world, and thus to build mathematical "intuition". The Geoscience course typically has a student enrollment of 180, while Math course enrollments are capped at 40 students/section.

In its most simplified form, Darcy's Law can be given as:

$Q = -KA\ h/L$ (Geoscience course) and $Q = K*A*i$ (Math course)

Where Q = discharge rate; K = hydraulic conductivity; A = cross-sectional area; h/L = the change in water table elevation/distance; and i = change in water table elevation/distance.

Perhaps more clearly represented to introductory students as:

Q = -KA(h1-h2)/(L), where h1 is the upslope elevation location, and h2 is the downslope equivalent. L represents the distance over which the change in h occurs. Note that the negative sign (in geologic applications) indicates that flow is in the direction of decreasing hydraulic head.

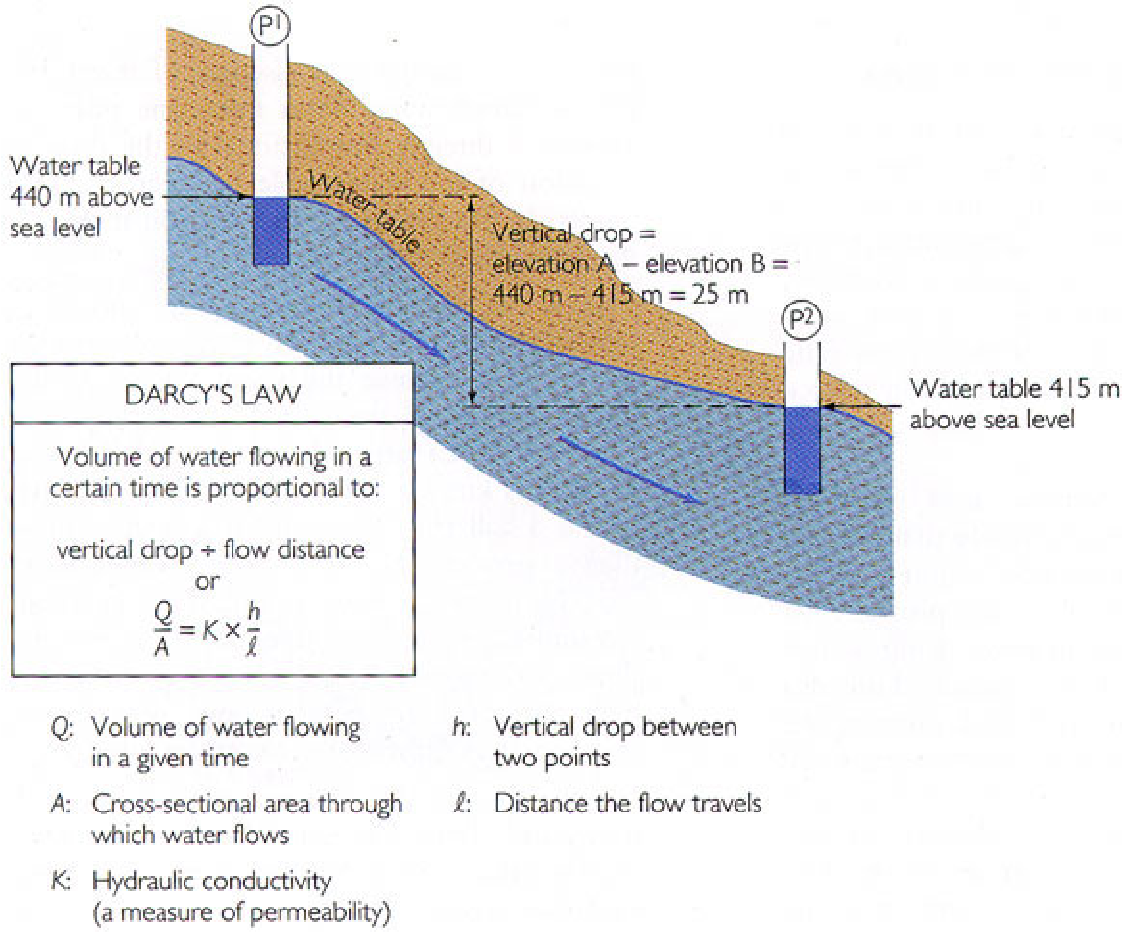
Darcy Variables		
Variable	**physical meaning**	**units**
Q	Discharge rate (volume of water across an area per unit time)	volume/time (cm3/s; gal/day)
K	Hydraulic Conductivity (measure of permeability)	length/time (cm/s)
A	Cross-sectional area through which water flows	length squared (cm2)
h	vertical change in water table elevation	length (m)
L	distance over which water table elevation changes	length (m)
i	hydraulic gradient (h/L) or slope of water table	length/length (unitless)

In the Introductory Geoscience course, the primary goal of presenting Darcy's Law is to help the students understand, in a relative sense, the variation of permeability with sediment size and sorting. We expect that by the end of this unit, students will understand that:

- Q is proportional to the cross sectional area of the flow (A).
- Q is proportional to change in hydraulic head (here represented by the change in water table elevation).
- Q is inversely proportional to L the length of the filter.
- K depends on grain size, sorting, and pore shape.

In addition, we anticipate that students will be able to evaluate a graphical representation of several aquifer materials and determine which material has the higher conductivity.

Geoscience presentation:
Given the following figure:



Outline parameters of equations for students, emphasizing that we are approximating the pressure surface as the water table surface in this example. Work through units of each variable in the equation.

In Winona Country, the hydraulic conductivity varies greatly, especially because of the occurrence of karst throughout the region. Conductivity values measured by the Minnesota Geological Survey indicate variation over 4 orders of magnitude in the Jordan-Praire deChein aquifer (in the Winona area, this is represented by the Jordan-Oneota lithologies visible along Garvin Heights Rd.). Values range from approximately 0.1 ft/day to over 1000 ft/day.

Given the conductivity value for a given unit in a particular region, it is possible to calculate both the horizontal and vertical hydraulic gradient.

Horizontal hydraulic gradient is simply the slope of the water table or potentiometric surface. It is the change in hydraulic head over the change in distance between the two monitoring wells or h/L.

In mathematical terms, horizontal gradient is rise over run.
h/L = difference in head / horizontal distance between wells = (h2 - h1) / L

Vertical hydraulic gradient is h/L = difference in head / vertical distance between wells = (h2 - h1) / (z2-z1)

Velocity of Groundwater Movement
Based on Darcy's work, we can estimate the velocity of water or how fast the water is moving between points. Velocity is calculated by using hydraulic conductivity, porosity, and hydraulic gradient.

V = (K / n) (h / L), where: n = porosity (a measure of the void space between grains).

Explaining the Above Concepts in a Quantitative Context
If we use some numbers, we can readily illustrate these concepts:

Problem 1
Data from three piezometers located within a few feet of each other is as follows:

	A	**B**	**C**
Elevation of Land Surface (Ft)	335	335	335
Depth of Monitoring Well (ft)	170	130	85
Depth to Water (ft below surface)	90	82	70

a. What is the hydraulic head (h) at each?
(surface elevation - depth to water)

EX. A=335-90=245 ft
B=
C=

b. What is the pressure head (hp) at each?
(depth of well - depth to water)

EX. A=170-90=80 ft
B=
C=

c. What is the elevation head (z) at each?
(land elevation - depth of well)

EX. A=335-170=165 ft
B=
C=

d. What is the vertical hydraulic gradient between Well A and Well B?
[(head B - head A) / (depth well A - depth well B)]

[(253-245) / (170-130)]=8/40 or 0.2

Problem 2
Two wells are located 100 feet apart in a sand aquifer with a hydraulic conductivity of 0.04 feet per day and 35% porosity. The head of well 1 is 96 feet and the head of well 2 is 99 feet.

a. What is the horizontal hydraulic gradient between the wells?
[(head 2 - head 1) / L

[(99-96) / (100)]=3/100 or 0.03

b. What is the velocity of water between the two wells?
V= (K / n) (h /L)

V= (0.04 ft/d / 0.35) (0.03)=0.0034 ft/d

Problem 3
Study the graph below, which represents variation in discharge with drop in head.
The slope of the line tells us something very important about the conductivity of the material.

A. Which line represents the aquifer material that more easily transmits water? Why?
B. Estimate the hydraulic conductivity for the aquifer material measured in each series.

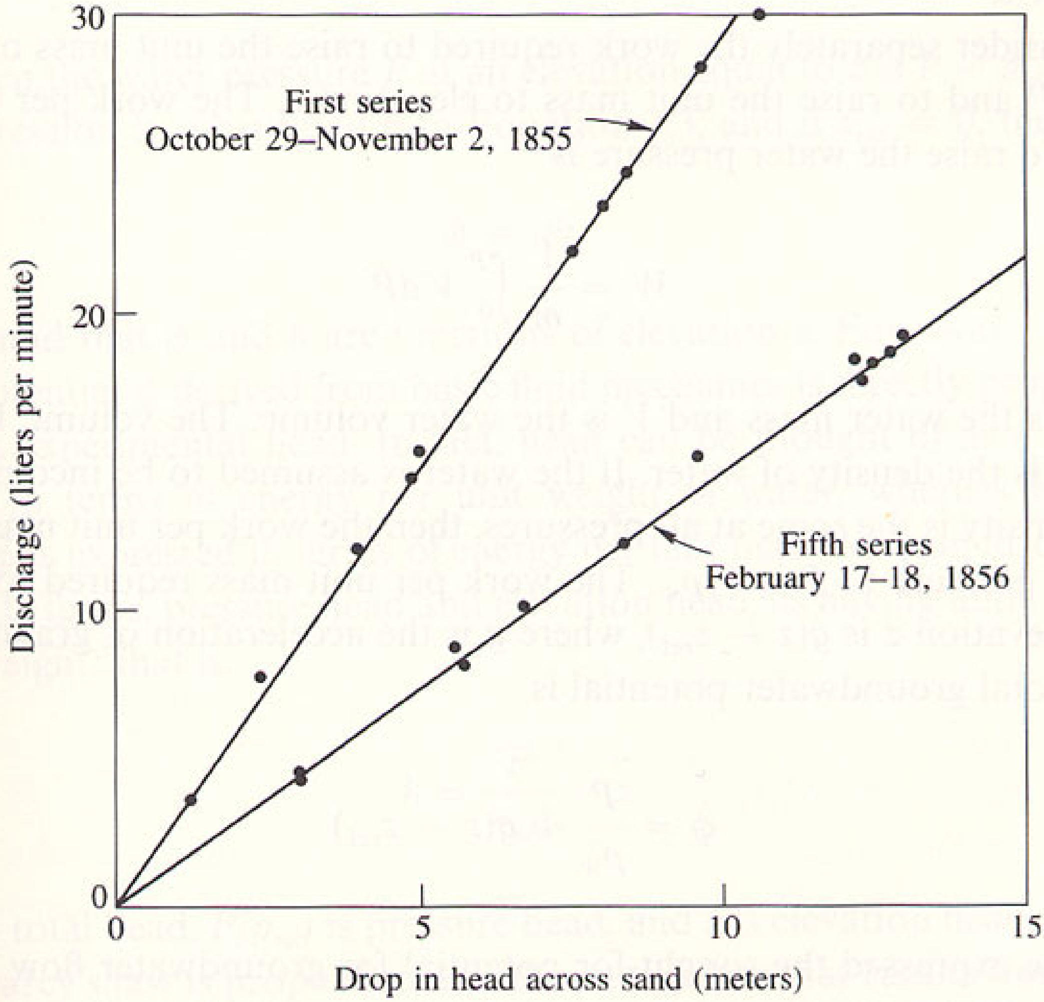


Figure 1.3
Darcy's data showing that discharge is proportional to head drop for two different sands. (From Hubbert, 1969. © 1956, Society of Petroleum Engineers of AIME, published *JPT*, Oct. 1956; *Trans.* AIME, 1956.)

Math Application:

Directions: Answer the following questions using Darcy's Law: $Q = K A h/L$

1. Given that the units of Q are in volume per time, determine the type of units (length, length squared, time, etc.) used for h, L, A, and K.

2. Fill in each blank below with either "increases" or "decreases." Then give an intuitive explanation of why the relationship is true.

(a) As the hydraulic conductivity increases, the discharge rate ________________.

(b) As the cross-sectional area decreases, the discharge rate ________________.

(c) As the head drop for a fixed length of flow decreases, the discharge rate ___________.

(d) As the length of flow for a fixed head drop increases, the discharge rate ___________.

3. Fill in each blank with either "directly proportional" or "inversely proportional."

(a) Discharge rate is ___________________________ to hydraulic conductivity.

(b) Discharge rate is ___________________________ to cross-sectional area.

(c) Discharge rate is ___________________________ to head drop.

(d) Discharge rate is ___________________________ to length of flow.

4. Find the effect on the discharge rate, as a ratio of the old discharge rate (i.e., calculate Q_new/Q_old), if all other values are held constant and:

(a) the cross-sectional area doubles;

(b) the head drop is tripled;

(c) the length of flow is doubled;

(d) the hydraulic conductivity is increased by 10%.

5. Calculate the total discharge rater for an aquifer in which:
the hydraulic conductivity is 90 feet per day;
the cross-sectional area is 50 ft. by 100 ft.;

the water table head drops from 76 ft. to 45 ft.; and
the length of the aquifer is 1000 ft.

6. Imagine an underground sand aquifer that is 20 ft. thick, 300 ft. wide, and has a hydraulic conductivity of 300 ft/day. Two test wells 500 feet apart have been drilled into the aquifer along the axis of flow, and the measured head values at these wells were found to be 120 and 98 feet respectively.

(a) Draw a clear diagram of the situation.

(b) Find the total discharge rate in a given cross-section of this aquifer.

7. Suppose that the aquifer described in problem #6 above is composed of gravel rather than sand, and has a hydraulic conductivity of 1500 ft/day (rather than 300 ft/day). Recalculate the total discharge rate Q_gravel. How does the discharge rate for the gravel medium compare to the discharge rate for the sand aquifer? (I.e., calculate explicitly the numerical value of the ratio Q_gravel / Q_sand.)

8. Suppose that the aquifer described in problem #6 above is composed of sandstone rather than sand, and has a hydraulic conductivity of 2 inches per year (rather than 300 ft/day). Recalculate the total discharge rate Q_sandstone. How does the discharge rate for the gravel medium compare to the discharge rate for the sand aquifer? (I.e., calculate explicitly the numerical value of the ratio Q_sandstone/ Q_sand.)

9. Use the figure below (Figure 2-14) below to answer the following questions.

(a) Calculate h_1 = the head at Well 1, and h_2 = the head at Well 2.

(b) Calculate the head drop h.

(c) Does the figure give you enough information to calculate the total discharge rate? If not, what additional information is needed?

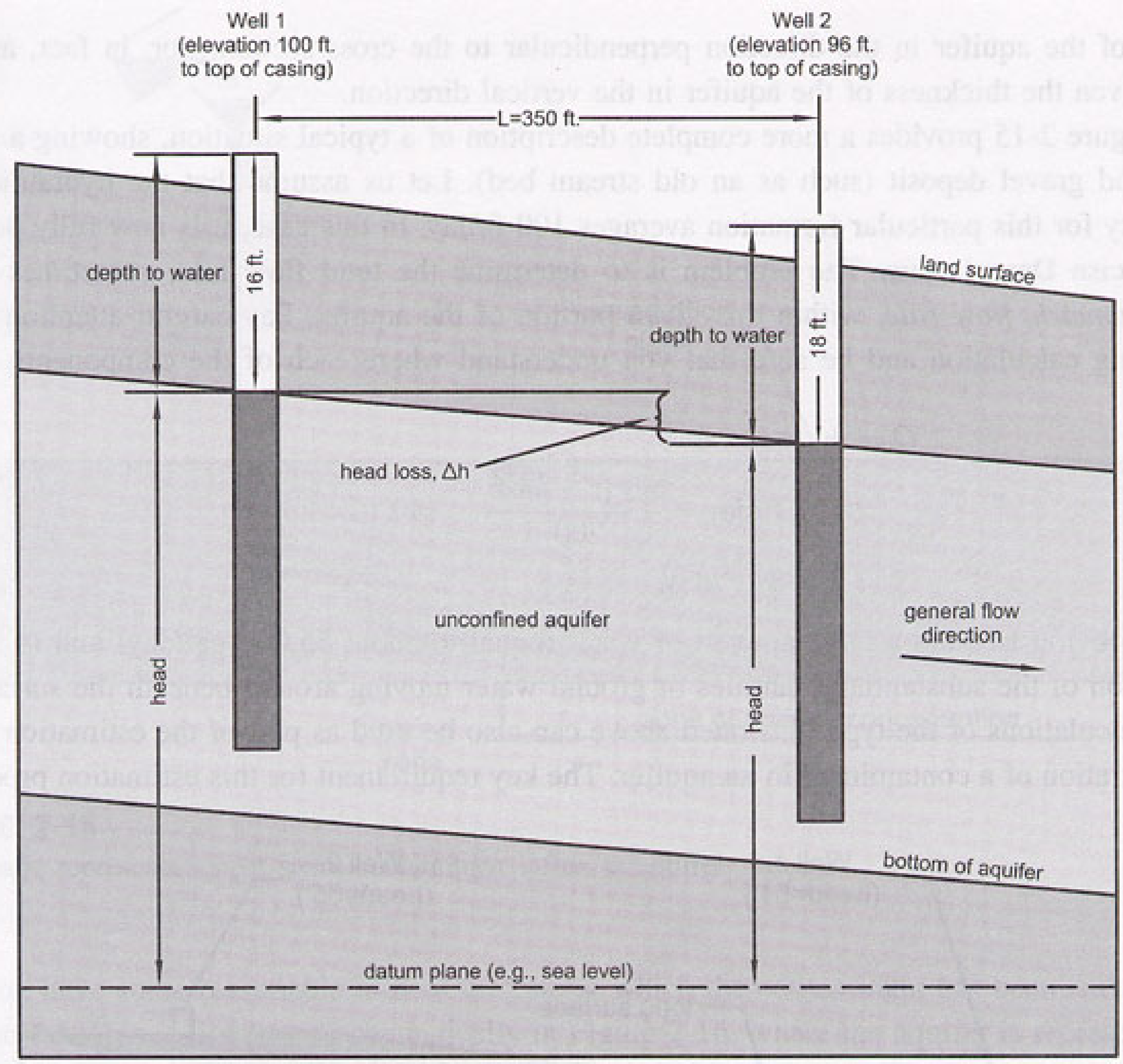


FIGURE 2-14
Section view of unconfined aquifer with two monitoring wells (not drawn to scale)

10. Suppose that the aquifer in the figure above (Figure 2-14) is composed of coarse porous sand with a hydraulic conductivity of 500 ft/day.

(a) Calculate the flow rate through any single imaginary planar surface of area one square foot, oriented perpendicular to the direction of flow.

(b) Describe how such a surface would be oriented with respect to the plane represented by Figure 2-14.