Scarp Diffusion Lab Readme

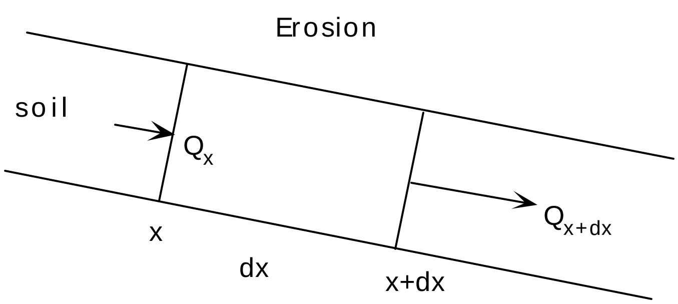
This modeling project is based on a problem of great geomorphological interest: how landscapes evolve over time in response to processes such as raindrop impacts, the annual cycle of freeze-thaw, tree throw, and the action of burrowing animals. All of these agents act to move soils and sediments from topographic high points to low points and are given the name diffusive processes because they tend to diffuse away sharp corners in the landscape. For example, the tops of initially sharp moraines tend to become rounded over time as erosion removes material from the moraine crest and deposits it along the base.

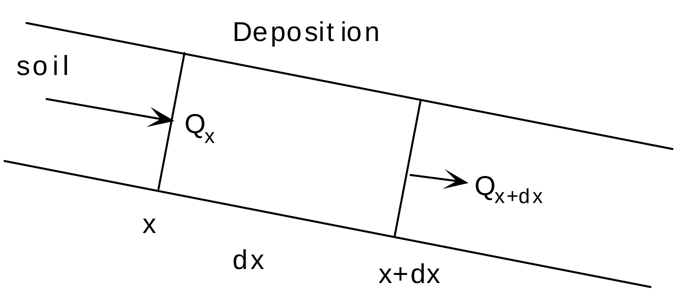
The rate at which material is moved, called the flux (Q), is directly proportional to the local slope of the topography (dz/dx):

$$Q = -a * \frac{dz}{dx} \qquad \text{Eqn. 1}$$

where α is a proportionality constant related to the number of raindrops striking per square meter over time, the density of animal burrows, or some other value reflective of the particular diffusive process at work.

We can divide any hillslope into a number of small boxes.





As diffusive processes operate, material enters the upper side of each box and leaves along the lower side of the box. We'll call these fluxes Q_x and Q_{x+dx}. If $Q_x < Q_{x+dx}$, more material leaves the box than enters it, resulting in erosion. If $Q_x > Q_{x+dx}$, the opposite is true and deposition occurs. The change in the flux over the distance of any small hillslope box, therefore, causes a change in the elevation of the box over time (dz/dt), a relationship that can be written as follows:

$$\frac{dz}{dt} = -\frac{1}{1-p} * \frac{(Q_{x+dx} - Q_x)}{dx} = -\frac{1}{1-p} * \frac{dQ}{dx} \qquad \text{Eqn. 2}$$

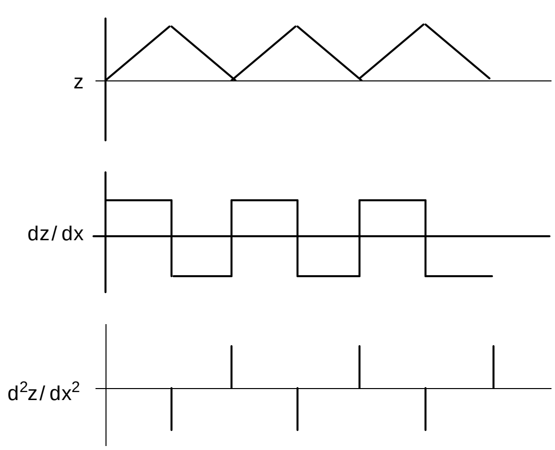
where p is the porosity of the sediment.

Substituting equation 1 for Q into equation 2 and differentiating with respect to x gives:

$$\frac{dz}{dt} = k * \frac{d^2z}{dx^2}$$

Where κ is $\alpha/(1-p)$ and is called the diffusivity.

The second derivative of z with respect to x is the curvature of the landscape, or change in slope with distance. This equation, called the "diffusion equation," says that the change in elevation of any hillslope element over time (dz/dt) depends on the curvature of the landscape (d^2z/dx^2). To understand the curvature term, it is helpful to consider what the first and second derivatives of z with respect to x are. Consider the following drawing of a hilly topography:

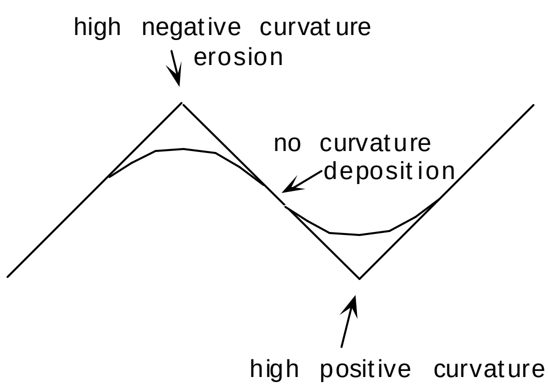


In the figure at the top, we have elevation (z) plotted as a function of distance (x). In the second figure, the slope is plotted as a function of distance. Note that the slope is initially constant and positive as we travel up the left flank of the first hill and then becomes constant and negative as we travel down the right flank of the hill. Successive hills show the same behavior. In the bottom figure, the curvature (or change in slope with distance) is plotted. Following the slope plot, we see that there's no change in slope with distance initially, so the curvature value is 0. At the point where the slope changes from positive to negative (i.e., the top of the hill), the curvature becomes highly negative. Curvature is then again zero as the slope becomes constant, and then becomes highly positive when the slope changes from negative to positive (i.e., the bottom of the first valley).

Since the diffusion equation relates the change in elevation within a hillslope box to the local curvature:

$$\frac{dz}{dt} = k * \frac{d^2z}{dx^2}$$

we can now predict where erosion and deposition will occur. On the flanks of hills, where the slope is constant, no change in elevation occurs with time because the curvature value is 0. At the crests of hills, the curvature is negative, and since κ is a positive constant, the change in elevation over time is negative (i.e., erosion). In valleys, the curvature is positive, so the change in elevation over time is positive (deposition).



In the modeling exercise I have prepared, I ask the students to apply these principles to a landscape that consists of two marine terrace platforms separated in elevation by a sea cliff.

Under Teaching Materials you will find the following:
1) Copies of the exercise for students in Adobe Acrobat (.pdf) format
2) Copies of the instructor answer key in Adobe Acrobat format
3) A STELLA model of scarp diffusion
4) A Fortran 90 model of scarp diffusion