EXPERIMENTAL MATH & EDUCATION

> "Mathematics does not grow through a monotonous increase of the number of indubitably established theorems but through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and refutations" Imred Lakatos

Mathematics has always had an intrinsic experimental component. This is perhaps more evident in the case of the empirical mathematical knowledge of ancient cultures. Mathematical results such as the Pythagorean theorem, know by the Babylonians more 1000 years before it was formally proven by Pythagoras, were usually obtained by a direct verification in special cases. Experimentation is also self evident in the case of computer aided mathematical findings. A good example is the discovery of the Lorenz attractor, an early example of a chaotic dynamical system, found by Edward Lorenz by investigating anomalous behaviors in a numerical weather model. In general, some sort of experimentation is intrinsic to the process of mathematical discovery. Before results are formulated as theorems, mathematicians are engaged in an intensive work of trial an error computations involving selected examples that illustrate the mathematical structure of the hypothetical results or conjectures. Working examples, expanded and re-dimensioned by modern technologies to include computer simulations and visualizations, play in this way a fundamental role in the process of mathematical inquires.

With the advent of digital computers and the development of powerful and sophisticate numerical and symbolic software, the experimental component of mathematics has been dramatically enhanced. Computers have transformed mathematical experimentation into a much more powerful and accessible tool not only for professional mathematicians but also for students.

After a dynamical interplay between experimentation and formalization, conjectures may eventually be proven and transformed into theorems. But sometimes they are refuted, or failed proofs exposed, by the discovery of a suitable counterexample. In a sense, examples and counterexamples conform the alpha and omega of experimental mathematics. Examples are not only instrumental at the beginning of a mathematical exploration, but they may also be important at the end of the process.

Experimental Mathematics is now recognized as a separate area of study. It has been defined as "a type of mathematical investigation in which computation is used to investigate mathematical structures and identify their fundamental properties and patterns. As in experimental science, experimental mathematics can be used to make mathematical predictions which can then be verified or falsified on the bases of additional computational experiments"[1].

In education, the possible uses of experimental mathematics are endless. With the aid of computer simulations students can develop their intuition about a mathematical object, and obtain a level of understanding that can serve as the basis for building a deeper knowledge. There is an interesting interplay between theoretical concepts and techniques, and their corresponding computer simulations. The more mathematics we know, the better use we can make of computer simulations, which in turn will enhance the understanding of a given mathematical problem. This dialectic interplay is similar to the interaction between theory and experimentation in the natural sciences. Computer simulations introduce the laboratory experience of the natural sciences into mathematics.

In the SERC teaching activities "Tracking Groundwater Pollution" and "Global Warming: A Zonal Energy Balance Model", developed for the workshop Teaching Geosciences with MATLAB, I combine the use of experimental mathematics with the SENCER approach to improve undergraduate STEM education by connecting learning to critical civic questions. For the present workshop I am using a similar strategy to develop the teaching activity "The More the Merrier in the Math of Population Ecology". In this activity students will use numerical simulations of differential equations to gain insight into some mathematical models and theorems related to habitat destruction and the role of dispersal in the survival of a population in danger of extinction.

[1] Weisstein, Eric W. "Experimental Mathematics." From *MathWorld*--A Wolfram Web Resource.