Linear Systems and Matrices

Name _____

Student ID #_____

Purpose: This assignment will test your knowledge of linear systems and matrices using basic Computer Algebra System (CAS) matrix/vector functions.

Format: Your session should begin with a header containing the date, project number, your name, and Student ID number.

Sample Session:

```
% October 12, 2007
% CAS Project 1
% Name:
% Student ID#:  1234567
% Problem 1.
%
A=[1 2 3;4 5 6;7 8 9]
```

or

```
A=
1 2 3
4 5 6
7 8 9
.............................
```

Notes: Comment your session well. If using MATLAB put `%` sign in the beginning of each comment line.

Problem 1. Consider the linear system:

$$\begin{aligned} x - 2y + 3z &= b_1 \\ -x + 3y &= b_2 \\ 2x - 5y + 5z &= b_3 \end{aligned}$$

Let b_1, b_2, and b_3 be the first three digits of your Student ID number. (For example: My Student ID# is 2349876. This makes $b_1 = 2$, $b_2 = 3$, and $b_3 = 4$.) Use the CAS to solve the system.

Problem 2. Use the CAS to determine which of the following matrices are row-equivalent to

$$X = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 9 & 10 & 11 & 12 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}, \quad C = \begin{bmatrix} 12 & 11 & 10 & 9 \\ 4 & 3 & 2 & 1 \\ 8 & 7 & 6 & 5 \end{bmatrix}$$

Problem 3. Enter the matrices

$$S = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & 1 & 2 \end{bmatrix} \qquad \text{and} \qquad T = \begin{bmatrix} -5 & 6 & 7 \\ 0 & -1 & 2 \\ 4 & 0 & -3 \end{bmatrix}$$

where matrix S is made of the digits of your Student ID#. Use CAS to find:

(a) $S + T$ (b) $4S - 5T$ (c) ST (d) $(TS)^T$

Problem 4. Write the following system of the linear equations in the form $AX = B$ and use CAS to solve the system.

$$\begin{aligned} 3x + 3y + 4z &= b_1 \\ x + y + 4z &= b_2 \\ 2x + 5y + 4z &= b_3 \end{aligned}$$

where b_1, b_2, and b_3 are the first three digits of your Student ID number.

Problem 5. Let $D = \begin{bmatrix} 1 & \frac{1}{8} \\ 0 & \frac{1}{10} \end{bmatrix}$. Use CAS to find D^2, D^7, and D^{15}.

Problem 6. Enter the matrices

$$F = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Use the CAS to form the block-matrix **M=[F G;G H]** and find the smallest value of n such that $M^n = M$.

Problem 7. Let S be the same 3×3 matrix as in the Problem 3. Use the CAS to find the inverse of S. Is S a singular matrix? Then adjoin a (3×3) identity matrix I to S to form the 3×6 matrix **L=[S I]**. Row-reduce L using CAS. (If using MATLAB row-redce with **rref** command) to compute the inverse of S again. What do you observe?

Problem 8. Let S and T be the same 3×3 matrices as in the Problem 3. Use the CAS to calculate

(a) $S^{-1}T^{-1}$ (b) $(ST)^{-1}$ (c) $(S^{-1}T)^T$ (d) $((TS)^T)^{-1}$.

Problem 9. Let S and T be the same 3×3 matrices as in the Problem 3. Use CAS to calculate the determinants of the following matrices:

(a) S (b) T (c) ST (d) $(ST)^{-1}$.

Problem 10. Let real number t be your birthday. (For example if your birthday is October 12, 1975 then $t = 12$.) Use CAS to calculate the determinant of the matrix:

$$A = \begin{bmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{bmatrix}$$

Does the value of the determinant depend on t? Explain your answer.