## GPS, Strain \& Earthquakes Unit 2: Physical models of strain extended student exercise

Vince Cronin (Baylor University) with context by Nancy West and Shelley Olds (EarthScope).

## Model A: Bungee cords

Please complete this worksheet to understand quantitatively how the length of a bungee cord changes as you pull on its ends. You know what will happen, of course, but you've probably never measured the changes. Why bother? The measured change in length will teach you principles of strain in its simplest form-one-dimension.

## Step 1. Assemble the apparatus.

Follow your instructor's instructions on setting up for the activity if the model is different than the one shown below (Figure 1) Otherwise, pull the loose bungee end to the near hook and place the ruler with the zero-end at the leftmost of the two black threads. Tape the ruler to the board.


Figure 1. An example of the bungee cord apparatus. Note the position of the ruler with respect to the black thread on the left side.

Step 2. Measure the original lengths-the initial state.
Follow your instructor's instructions regarding safety.
Distance to black thread ( $l_{\text {oblack }}$ ) = $\qquad$ cm Distance to red thread $\left(l_{o r e d}\right)=$ $\qquad$ cm

Step 3. Stretch the bungee to the second hook and measure the final lengths. Distance to black thread $\left(l_{f \text { black }}\right)=$ $\qquad$ $\mathrm{cm} \quad$ Distance to red thread $\left(l_{\text {fred }}\right)=$ $\qquad$ cm

## Step 4. Calculate the extension.

Calculate the extension measured to the black thread. $l_{o}$ is the original length. $l_{f}$ is the length after straining.
extension, $e$, is $\frac{l_{f}-l_{o}}{l_{o}}$ $\qquad$
Calculate the extension measured to the red thread.
extension, $e$, is $\frac{l_{f}-l_{o}}{l_{o}}$

$$
e_{\text {red }}=
$$

## Step 5. Compare the extension.

Discuss how the extension measured using the black thread compares to that using the red one.

## Step 6. Meet displacement vectors.

Geodesists measure changes in the shape of the crust with GPS units that can detect movement as little as a few millimeters per year. You can see and use this data from the GAGE GPS Velocity Viewer (http://www.unavco.org/software/visualization/GPS-Velocity-Viewer/GPS-Velocity-Viewer.html). The data appears as an arrow in which the length of the arrow corresponds to the distance the station has moved and the direction it points, obviously, shows the direction it has moved. It is a "displacement vector". (Figure 2)


Figure 2. Displacement vectors for GPS stations in California.

In order to understand two-dimensional strain on Earth later, let's think about the bungee in terms of vectors. Think of the bungee in a coordinate system lying along the x -axis. Imagine the position of the black (or red thread) to be shown as a "location vector" which starts at the zero-mark of the ruler and goes along the x -axis to the thread. (Alternatively, you can think of the position as a coordinate along the x -axis.) The original position of the black thread was the distance $l_{o}$ black from the origin. As a vector, we would call the same spot $x_{b o}$ ( $x$ for the $x$-axis, $b$ for black, and $o$ for original).
a. Return the bungee's hook to the pin on the left. Slip a piece of paper under the bungee and mark the position of the black thread on one side of the bungee and the red thread on the other side.
b. Now extend the bungee (again) by hooking it to the pin on the right. On the paper, mark the new position of the black and red threads.
c. Draw arrows on the paper showing the change in position of both the black and red threads. These arrows are displacement vectors. Designate them $u_{\text {black }}$ and $u_{\text {red }}$.
d. Measure the displacement vectors.
length $u_{\text {black }}=$ $\qquad$ cm
length $u_{\text {red }}=$ $\qquad$ cm
e. Calculate extension again as the difference between the lengths of the displacement vectors divided by the difference between the lengths of the initial location vectors.

$$
e=\frac{u_{r e d}-u_{\text {black }}}{x_{r o}-x_{b o}}=\frac{\Delta u}{\Delta x}=
$$

f. How does $e$ measured in step 6e compare to $e$ measured in step 4?

## Model B: Compressional springs

## Name(s):

$\qquad$
Please complete this worksheet to understand quantitatively how the length of a compressional spring changes as you push the ends together. Recall a geologist's definition of extension, $e$, from your exploration of bungee cords.

## Step 1 (Optional). Design your apparatus.

For your compressional spring, design a way to measure extension, $e$, under at least two conditions. Draw the design of your apparatus.

Describe the two conditions.


Figure 1: Cavallini da gioco. (Italian playground horses.) Photograph by Lucarelli. 2009.

List materials you need.

Check your design and conditions with your instructor. Instructor's initials: $\qquad$

Step 2. Build and test your apparatus.
Follow your instructor's instructions regarding safety.

Step 3 (Optional). Redesign your apparatus.
Describe changes needed and why.

List materials needed.

Step 4. If needed, rebuild and test your apparatus again.
Repeat until you are satisfied with the way your apparatus works.

## Step 5. Measure extension.

Under the first set of conditions, measure the original length and the shortened length. Repeat three times. Record your data.
Trial 1: Original length: ___ $\mathrm{cm} \quad$ Final length: ___ cm
Trial 2: Original length: ___ $\mathrm{cm} \quad$ Final length: ___ cm
Trial 3: Original length: ___ $\mathrm{cm} \quad$ Final length: ___ cm

Under the second set of conditions, measure the original length and the shortened length. Repeat three times. Record your data.

Trial 1: Original length: ___ $\mathrm{cm} \quad$ Final length: ___ cm
Trial 2: Original length: ___ $\mathrm{cm} \quad$ Final length: ___ cm
Trial 3: Original length: ___ $\mathrm{cm} \quad$ Final length: ___ cm

## Step 5. Calculate extension.

For both sets of conditions, calculate extension for each measurement (six total). Average for the first set and again for the second set.

Condition 1:
Trial $1 e=$

Trial $2 e=$

Trial $3 e=$

Average $e=$

## Step 5. Calculate stretch.

For both averages, calculate stretch, $S . S=1+e$.

Condition 1: Average $S=$

Condition 2:
Trial $1 e=$

Trial $2 e=$

Trial $3 e=$

Average $e=$

## Model C: Silly Putty ${ }^{\circledR}$

## Name(s):

Please complete this worksheet to experience pure strain first-hand. Strain is the change in shape or volume of an object. Your task is to play with Silly Putty ${ }^{\circledR}$ and to see what happens to it over time.

## Step 1. Prepare the Silly Putty ${ }^{\circledR}$.

Roll your handful of Silly Putty ${ }^{\circledR}$ into a fat cylinder. Flatten the ends and one side. Push the open end of a vial or small bottle into the flat side so that the putty has an indented circle. Follow your instructor's instructions regarding safety.

Measure the diameter of the circle in centimeters. Diameter $=$ $\qquad$ cm

## Step 2. Wait and observe.

As you do some other task, watch the Silly Putty ${ }^{\circledR}$. Note what you see happening to it. Write your observations.

## Step 3. Finalize your observations.

Sketch what the entire cylinder looks like. Pay particular attention to the indented shape. Also, draw the indented shape at a scale of $1: 1$.

Measure the long axis of the indentation and an axis perpendicular to it.
Long axis $=$ $\qquad$ cm

Perpendicular axis $=$ $\qquad$ cm

## Step 4. Analyze your observations.

Discuss what caused the Silly Putty ${ }^{\circledR}$ to change shape and illustrate your response with a sketch. This style of deformation is due to pure strain.

## Model D: Cards

## Name(s):

$\qquad$

Please complete this worksheet to experience simple strain first-hand. Your task is to shear a stack of cards with a circle and to note carefully what happens to the shape of a circle drawn on the stack's side.
Step 1. Prepare the cards.
Draw a circle on the edge of a stack of cards to look like that below. (Figure 3)


Figure 3. Deck of cards ready to shear.

Step 2. Shear the cards.
Using a book or the table, push against the side of the stack so that each card moves a little to the left of the one below it. This is called "shearing", and the resulting change of shape is simple strain. (Figure 4)


Figure 4. Deck of cards sheared against a table.

## Step 3. Draw and measure strain axes.

Use your red pen and a straight edge to draw an axis through the longest part of the ellipse showing on the side of the cards. This is the "major axis" of the strain ellipse. Use the blue pen
 to draw a shorter axis perpendicular to the red one. This one is the "minor axis" of the strain ellipse. (Figure 3)

Figure 3. Strain axes drawn through strain ellipse.

Put the stack flat on the table and measure the angle between the major axis of the ellipse and the table.

Angle = $\qquad$ degrees

## Step 4. Reverse the shear.

Restore the deck of cards to its original shape and so that the ellipse has reverted to a circle.
What is the angle between the major and minor axes?
Angle = $\qquad$ degrees

Again, put the stack on a table. What is the angle between the major axis and the table now? (Measure this.)
Angle = $\qquad$ degrees

How much did the major axis rotate from the sheared deck to the restored deck?
Rotation $=$ $\qquad$ degrees

Now think of that rotation in reverse. From the initial state (or restored state), how much did the major axis rotate to its sheared state? Was the rotation clockwise or counterclockwise? This rotation is designated with an omega, $\Omega$.
$\Omega=$ $\qquad$ degrees (Clockwise or counterclockwise?)

This particular style of strain-with rotation of strain axes-is called simple strain.

