

## GPS Strain & Earthquakes Unit 2: Physical models of strain – basic student exercise

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### Model 1: Rubber Bands

Name(s): \_\_\_\_\_

*Please complete this worksheet to understand quantitatively how the length of a rubber band changes as you pull on its ends. You know what will happen, of course, but you've probably never measured the changes. Why bother? The measured change in length will teach you principles of strain in its simplest form—one-dimension.*

#### Step 1. Assemble the apparatus.

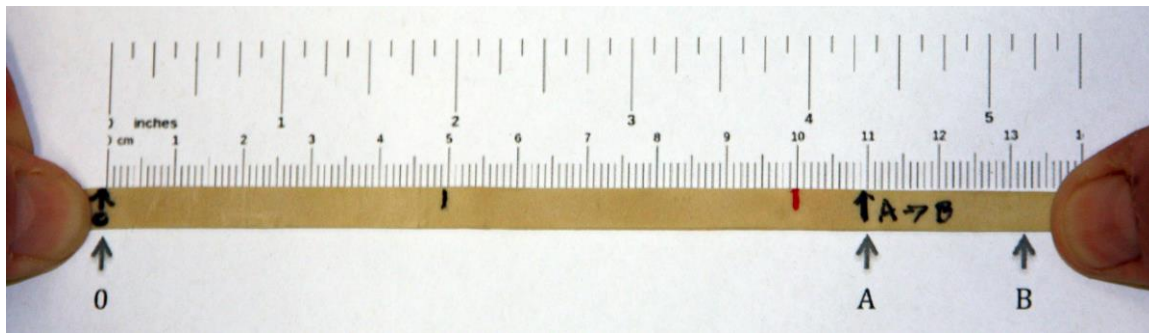


Figure 1. An example of the rubber band apparatus. Note the position of the ruler with respect to the zero and A-B arrows.

Use your thumb pin the zero arrow on the rubber band to the zero arrow on the ruler below (Fig. 1). Next stretch the rubber band so that the arrow labeled A -> B is aligned with the A arrow below the ruler.

#### Step 2. Measure the original lengths—the initial state.

Distance to black mark ( $l_{o\text{ black}}$ ) = \_\_\_\_\_ cm    Distance to red mark ( $l_{o\text{ red}}$ ) = \_\_\_\_\_ cm

#### Step 3. Stretch the rubber band so that the A-B mark on the band is aligned with the B mark below the ruler and measure the final lengths.

Distance to black mark ( $l_{f\text{ black}}$ ) = \_\_\_\_\_ cm    Distance to red mark ( $l_{f\text{ red}}$ ) = \_\_\_\_\_ cm

#### Step 4. Calculate the extension.

Calculate the extension measured to the black mark.  $l_o$  is the original length.  $l_f$  is the length after straining.

extension,  $e$ , is  $\frac{l_f - l_o}{l_o}$

$e_{\text{black}} =$  \_\_\_\_\_

Calculate the extension measured to the red mark.

extension,  $e$ , is  $\frac{l_f - l_o}{l_o}$

$e_{red} =$  \_\_\_\_\_

### Step 5. Compare the extension.

Discuss how the extension measured using the black mark compares to that using the red one.

### Step 6. Meet displacement vectors.

Geodesists measure changes in the shape of the crust with GPS units that can detect movement as little as a few millimeters per year. You can see and use this data from the GAGE GPS Velocity Viewer (<http://www.unavco.org/software/visualization/GPS-Velocity-Viewer/GPS-Velocity-Viewer.html>). The data appears as an arrow in which the length of the arrow corresponds to the distance the station has moved and the direction it points, obviously, shows the direction it has moved. It is a “displacement vector”. (Fig. 2.)



Figure 2.  
Displacement  
vectors for GPS  
stations in California.

In order to understand two-dimensional strain on Earth later, let's think about the rubber band in terms of vectors. Think of the rubber band in a coordinate system lying along the x-axis. Imagine the position of the black (or red mark) to be shown as a “location vector” which starts at the zero-mark of the ruler and goes along the x-axis to the mark. (Alternatively, you can think of the position as a coordinate along the x-axis.) The original position of the black mark was the distance  $l_{o\text{ black}}$  from the origin. As a vector, we would call the same spot  $x_{bo}$  ( $x$  for the x-axis,  $b$  for black, and  $o$  for original).

- Return the A-B arrow on the rubber band to the A arrow on the paper. Mark the position of the black mark on one side of the rubber band and the red mark on the other side.
- Now extend the rubber band (again) by stretching it so that the A-B arrow is aligned with the B arrow. On the paper, mark the new position of the black and red marks.
- Draw arrows on the paper showing the change in position of both the black and red marks. These arrows are displacement vectors. Designate them  $u_{black}$  and  $u_{red}$ .
- Measure the displacement vectors.

length  $u_{black} =$  \_\_\_\_\_ cm

length  $u_{red} =$  \_\_\_\_\_ cm

- Calculate extension again as the difference between the lengths of the displacement vectors divided by the difference between the lengths of the initial location vectors.

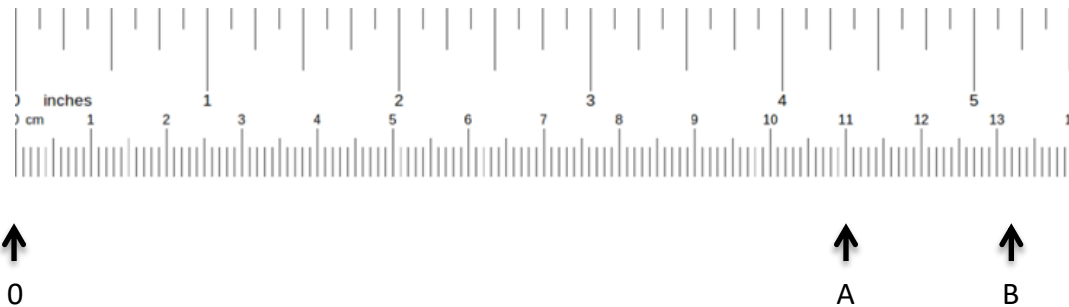
$$e = \frac{u_{red} - u_{black}}{x_{ro} - x_{bo}} = \frac{\Delta u}{\Delta x} =$$

- How does  $e$  measured in step 6e compare to  $e$  measured in step 4?

**Step 7. Translate the rubber band**

Repeat the procedure from step 6, but instead of stretching the band as we have before, move the zero arrow to the 2cm point and the A-B arrow to 13 cm. Draw the arrows and measure displacements as before.

g. What is the extension in this case?

**Experimental Apparatus**

**Model 2: T-shirts**

Name(s): \_\_\_\_\_

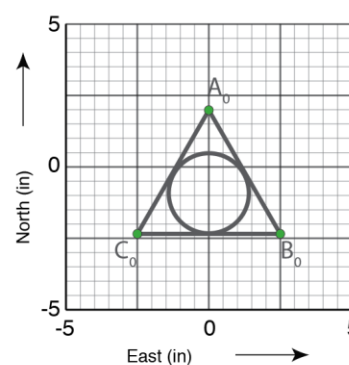
*Please complete this worksheet to understand how deformation plays out in two dimensions. This will include moving a piece of tee shirt fabric as a whole and stretching it in various ways. Why? This introduces the changes in position and shape of a region of Earth's crust. The tee shirt will represent an area defined by three GPS stations, each of which could move over time. We will draw displacement vectors for each corner of a triangle, representing our three GPS stations (A, B, C,) as well as sketch the change in shape of the triangle and an inscribed circle to visualize the strain in the area between the stations.*

Set up. Gather your team and grab the shirt from each of the three corners so that the fabric is taut and the circle is still round.

Step 1. Translation.

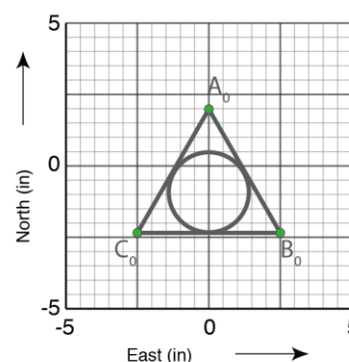
Move together as a group toward the north (defined by the A vertex) ~3 inches.

Sketch the displacement vectors for each apex of the triangle and the deformed shape of the initial triangle and circle (already drawn for you) on the graph at right.

Step 2. Rotation.

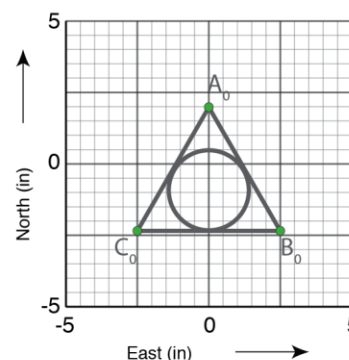
Move together as a group, keeping the circle round, ~1/6 of a full rotation (2 hours on a clock face or 60 degrees).

Sketch the displacement vectors for each apex of the triangle and the deformed shape of the initial triangle and circle on the graph at right.

Step 3. Dilation.

Start again with the tee shirt taut and with circle round. Have all group members move away from the center of the circle ~ 3 inches.

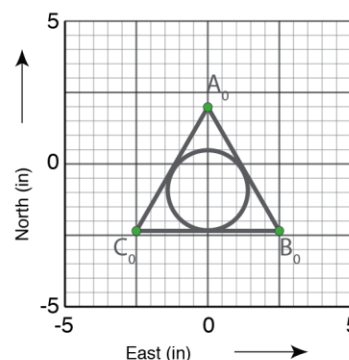
Sketch the displacement vectors for each apex of the triangle and the deformed shape of the initial triangle and circle on the graph at right.



**Step 4. Extension in one direction**

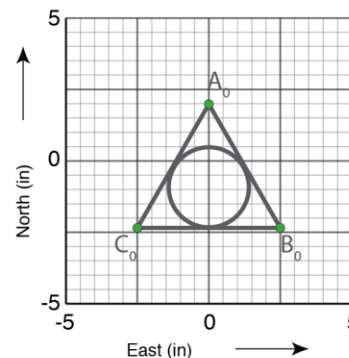
Start again with the tee shirt taut and with circle round. Have the person near vertex  $A_0$  move away from the other group members ~3 inches.

Sketch the displacement vectors for each apex of the triangle and the deformed shape of the initial triangle and circle on the graph at right.

**Step 5. Simple Shear**

Start again with the tee shirt taut and with circle round. Have the person near vertex  $B_0$  move parallel to the line between  $A_0$  and  $C_0$  ~3 inches.

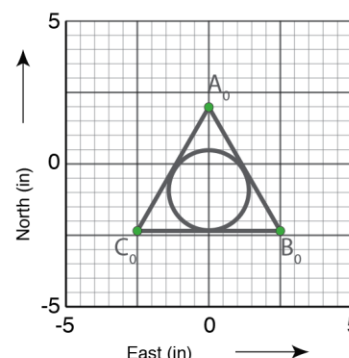
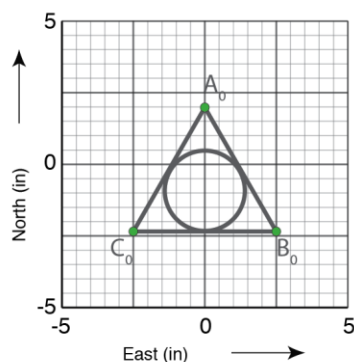
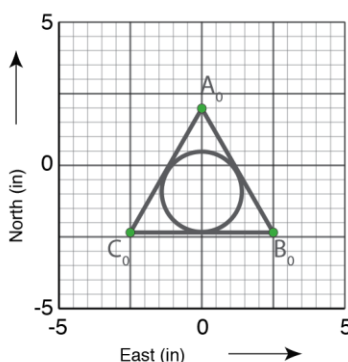
Sketch the displacement vectors for each apex of the triangle and the deformed shape of the initial triangle and circle on the graph at right.

**Step 6. General Deformation**

Start again with the tee shirt taut and with circle round. Each member of the group should move in any direction they want ~3 inches.

Sketch the displacement vectors for each apex of the triangle and the deformed shape of the initial triangle and circle on the graph at right.

Describe your deformation as the addition/subtraction of the basic components we described above.

**Translation****+Rotation****+Strain**