# Big Players in a Small Market 

# Pure and Mixed-Strategy Nash Equilibria in Stylized Market Participation Decisions 

Instructor Solutions

It is important to note that filling out the matrix is not difficult, but not trivial. It has to do with the fact that both companies start with $\$ 1 \mathrm{M}$ each. Once that money is spent, the revenue is something which goes into the payoff matrix (NOT revenue minus the cost of construction). Firms strategies are labeled $E$ for "Enter" and $N E$ for "Not Enter". Depending on the preferences in town, $(E, E),(E, N E)$, and $(N E, E)$ can all be Nash equilibria in pure strategies. Note an interesting fact that both firms not entering the market is NEVER the option in this activity.


Figure 1: Even revenue split. Entering is a dominant strategy for both firms.

> |  |  | Chipotle |  |
| :---: | :---: | :---: | :---: |
|  |  | $E$ |  |
| Qdoba | $E$ | $N E$ |  |
|  | $0.75,1.75$ | $2.5,1$ |  |
|  | $1,2.5$ | 1,1 |  |

Figure 2: Preference for Chipotle. Entering is dominant for Chipotle. $(N E, E)$ is unique Nash.


Figure 3: Preference for Qdoba. Entering is dominant for Qdoba. $(E, N E)$ is unique Nash.

Introduction of the extra cost of construction eats into the revenue if both firms enter. Here both $(E, N E)$ and $(N E, E)$ are Nash equilibria, but we cannot know which one is the actual "resolution" of this game.

|  |  | Chipotle |  |
| :---: | :---: | :---: | :---: |
|  |  | $E$ |  |
|  |  | $N E$ |  |
| Qdoba |  | $0.75,0.75$ |  |
|  |  | $2.5,1$ |  |
|  |  | $1,2.5$ |  |
|  |  |  |  |

Figure 4: Preference for Qdoba. Entering is dominant for Qdoba. $(E, N E)$ is unique Nash.
Both firms can play a Nash in mixed strategies. Let $p$ be the probability of playing $E$.

$$
\begin{array}{r}
0.75 p+2.5(1-p)=p+(1-p) \\
0.75 p+2.5-2.5 p=1 \\
1.75 p=1.5 \\
p=0.857
\end{array}
$$

So both firms will play "Enter" with probability of 0.857 and "Not Enter" with probability 0.143 .

