

Teaching Structural Geology, Geophysics, and Tectonics in the 21st Century

Strain Analysis and Hyperboloidal Projections

F.W. Vollmer, 2012

Notes to the Instructor and Reviewer

The attached handout is for a structural geology laboratory that introduces students to basic concepts of strain analysis in structural geology. Many basic concepts are not presented in the handout, as they are covered in the text. Mainly new concepts are discussed, including an analytical solution to the Wellman digram and *hyperboloidal projections*, of which familiar Elliott and $Rf-\Phi$ diagrams are a subset. This latter allows a new way of thinking about strain graphs as projections. Teaching is done interactively using the program EllipseFit 2.

Note that this material is new, and my handout is *very* rough. The new concepts are currently only published as abstracts (see references) and are *in preparation* for publication, so please respect them as such. The program EllipseFit 2 is an update of EllipseFit 1 written in the 1980's, which in turn was based in part on programs beginning in the 1970's under W.D. Means. EllipseFit 2 has been entirely updated, using current, as well as new unpublished techniques. It is designed as an integrated package to include necessary techniques required for advanced field studies, through 3D ellipsoid fitting, but also for interactive use with undergraduate students in a structural geology laboratory.

Any use of this material should be acknowledged. Note that much of this material is still *in preparation* for publication. References are below for analytical solution of the Wellman digram (Vollmer 2011a), comparison of strain analysis methods (Vollmer 2011a), automatic hyperboloidal contouring (Vollmer, 2011b), and the EllipFit program (Vollmer, 2010). Reference to the contained material should be similar to:

Vollmer, F.W., 2012. Strain Analysis and Hyperboloidal Projections. Teaching Structural Geology, Geophysics, and Tectonics in the 21st Century. On the Cutting Edge (<https://serc.carleton.edu/NAGTWorkshops/structure/SGT2012/>).

Strain Analysis and Hyperboloidal Projections

Strain Analysis

Goals and Objectives

Structural geologists use numerous tools to unravel the deformation history of rock bodies, from thin sections to mountain belts. An important area of study is the determination of *strain* at various scales. Our first goal is to learn some common two-dimensional techniques for this. In order to do this on natural materials, various assumptions must be made; a second goal is to understand these assumptions. Generally, it must be assumed that the rock has some characteristic that defined a circle in the undeformed state, and an ellipse in the deformed state. Figure 1 shows the approximations of grains from an oolitic limestone as ellipses after deformation, and their assumed state after *unstraining* using the calculated strain. An assumption in this case is that the ellipses before deformation, when averaged, gave a circle. This section gives a brief summary of three techniques, which derive strain from, respectively: points, lines and ellipses.

In the following section, on *hyperboloidal projections*, we discuss projections to display two-dimensional strain data using methods analogous to the *spherical projections* studied earlier. All figures except 5 and 7 were created with EllipseFit (Vollmer, 2010).

Center-to-Center Methods

Center-to-center methods are commonly referred to as the *Fry method*, but include variations on the basic technique (Fry, 1979; Ramsay and Huber, 1983; Erslev, 1988; Erslev and Ge, 1990). These consider distributions of *points*, particle centers. Figure 2 shows examples of these graphs created using EllipseFit. Center-to-center methods assume that the points are *anticlustered*, which means that in the undeformed state each particle's center, *on the average*, will be the same distance from an adjacent particle's center. This condition is generally reasonable in sandstones made up of well-rounded equidimensional grains, although three-dimensional packing must be considered. Other candidates for this type of analysis include, for example, porphyritic igneous rocks in which nucleation is controlled by chemical potential, giving equally spaced phenocrysts.

Wellman Method

The Wellman method can be applied to objects in which initially perpendicular *lines* can be identified, such as brachiopod hinge and medial lines (Wellman, 1962). For brachiopods not parallel to a principal strain, this angle will be distorted by shear strain. Wellman's graphical technique is illustrated in most structural geology laboratory manuals (e.g., Ragan, 2009). An analytical solution (Vollmer, 2011a), is implemented in EllipseFit.

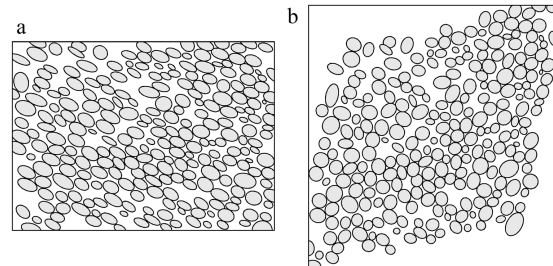


Figure 1. Deformed and undeformed ellipses from ooid grains.

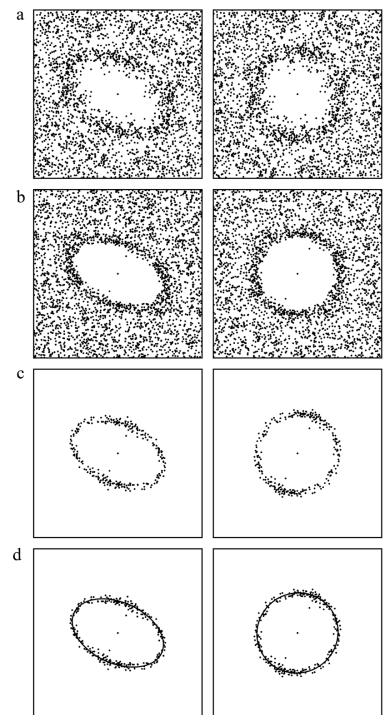


Figure 2. Center-to-center graphs, including Fry, normalized Fry, and enhanced normalized Fry, and with best-fit ellipse.

Eigenvectors and Numerical Methods

Strain ellipses are commonly given by the axial ratio, R , and angle from the x-axis, ϕ . If we consider a population of such *ellipses*, then an “average” or “best-fit” ellipse is desired. As we previously discussed, such “best-fit” values can be found using *eigenvectors*. A standard technique for determining the a best-fit strain ellipse from a set of ellipses uses shape-matrix eigenvectors (Shimamoto and Ikeda, 1976). Two related methods are *mean radial length* (Mulchrone et al., 2003), and *hyperboloidal vector mean* (Yamaji, 2008). These are each implemented separately in EllipseFit, however the calculation in each case gives identical results (Vollmer, 2010). Calculation of the three-dimensional strain ellipsoid can also be done using eigenvector methods (Shimamoto and Ikeda, 1976; Shan, 2008). Shan's method (Shan, 2008) is implemented in EllipseFit.

Hyperboloidal Projections

Yamaji (2008) provided a unifying parameter space for two-dimensional strain by applying concepts from hyperbolic geometry. This parameter space suggests new techniques for strain analysis, including the use of hyperboloidal equal-area and stereographic projections, and contouring of strain data on the unit hyperboloid, similar to techniques for contouring orientation data on a unit sphere (Vollmer, 2011b). Properties of standard strain graphs, such as the Elliott polar graph (Elliott, 1970) and the Rf- Φ graph (Dunnet, 1969), can be understood in terms of hyperboloidal projections, and it allows construction of automated reproducible contours for descriptive statistical analysis (Vollmer, 2011b).

Figure 4 shows examples of hyperboloidal projections, including the Elliot graph, an equidistant azimuthal projection (Figure 4a), hyperboloidal stereographic projection (Figure 4b), and an Rf- ϕ graph, a cylindrical projection (Figure 4c). These are shown in strained ($R = 4$) and unstrained states.

Azimuthal Hyperboloidal Projections

Azimuthal hyperboloidal projections are projections of the unit hyperboloid onto a plane (Figure 5). The origin of hyperboloidal is $C = (1, 0, 0)^T$, which corresponds to a strain of $R = 1$. In these projections azimuths, which correspond to values of 2ϕ , are preserved. The distance from the center of the azimuthal projection is a function of R , and strain is a rotation on the hyperboloid.

The *equidistant hyperboloidal projection* (Figure 4a and 6a) preserves distance radially, so the natural strain magnitudes, $\epsilon = 1/2 \log R$, are undistorted. Note that because of this property contour lines are distorted, but remain approximately concentric with increasing strain. The *equal-area hyperboloidal projection* (Figure 6c), in contrast, preserves area, but distorts ϵ . By analogy with spherical equal-area projections, this is useful for comparing relative *densities* of strain data sets.

The *stereographic (equal-angle) hyperboloidal projection* (Figure 4b and 6b) preserves angles, and hyperbolic curves plot as circles (Reynolds, 1993). Curves of equal distance from the origin are circles and, as distance on the hyperboloid is invariant with strain (strain is a rotation), the circles remain circles with strain. For a radially symmetric distribution, the *centroid* (planar average point) of the projected data plots at the same location as the best-fit ellipse.

The centroid of the projected data coincides with the best-fit strain ellipse only in the unstrained state, and on the stereographic projection. On other projections the centroid is not a good estimator of the applied strain. In evaluating strain ellipse data sets, the symmetry of the distribution is an important characteristic (Elliott, 1970; Dunnet and Siddans, 1971). Automatic contouring of the data gives an unbiased view of densities and symmetries. The equal-area projection provides an undistorted view of data *density*, while the equidistant projection maintains radial

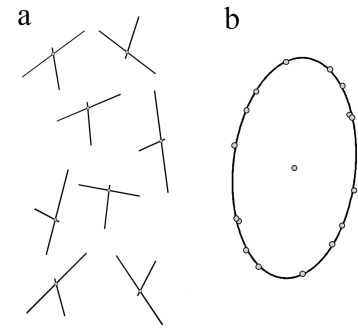


Figure 3: Concept of the Wellman method, with analytical solution.

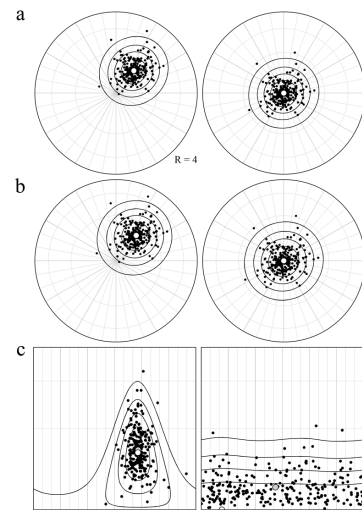


Figure 4: Hyperboloidal projections of strain data. See text for discussion.

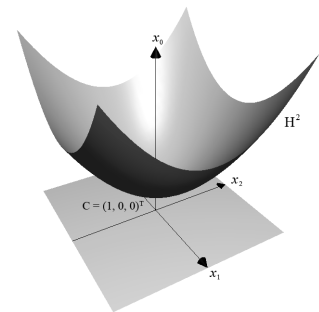


Figure 5: Principle of azimuthal hyperboloidal projections.

symmetry. The stereographic projection has the useful property that circular contours remain circular with strain, allowing a simple visual test for bimodal or multimodal prestrain fabrics (Vollmer, 2011b).

Cylindrical Hyperboloidal Projections

Cylindrical hyperboloidal projections project the unit hyperboloid onto the surface of a cylinder, which is then “unwrapped” and viewed on a plane (Figure 7). Figure 8 shows several examples, including *equidistant*, or logarithmic Rf- Φ (Figure 8a), and exponential, or linear Rf- Φ . Rf- Φ graphs (Figure 8c) which are widely used in strain analysis (e.g., Ramsay and Huber, 1983). Understanding them as hyperboloidal projections unifies the various strain graphs, and allows unbiased automatic contouring of data.

By analogy, recall that a Mercator projection is a cylindrical projection of a sphere. On a Mercator projection of the earth, each pole maps to a line, causing distortion at high latitudes. Similarly, a strain of $R = 1$, which plots at the hyperboloidal origin C , maps to a line, causing distortion at low strain values, and the centroid is not well correlated with the strain.

Figure 8 shows cylindrical hyperboloidal projections of initially uniform distributions to $R = 2$ and $R = 4$. Compare to Figure 6, which shows the same data on azimuthal projections. Note that the contours are identical for equal values of strain in the two diagrams, as they are calculated on the hyperboloid, changes in shape are due only to properties of the projection.

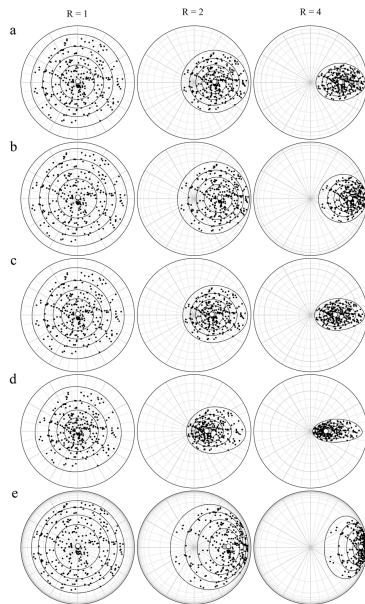


Figure 6. Equidistant, stereographic, equal-area, orthographic and gnomonic hyperboloidal projections of initially uniform distributions.

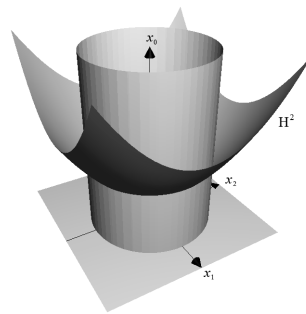


Figure 7. Principle of cylindrical hyperboloidal projections.

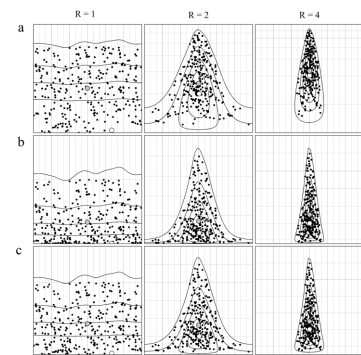


Figure 8. Hyperboloidal cylindrical projections: (a) equidistant (logarithmic Rf- Φ), (b) radial, and (c) exponential (linear Rf- Φ).

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