SUNY New Paltz
Department of Geology
Structure and Tectonics

Teaching Structural Geology, Geophysics, and Tectonics in the 21st Century

Spherical Projections and Data Analysis

F.W. Vollmer, 2012

Notes to the Instructor and Reviewer

The attached handout "*Spherical Projections*" and laboratory "*Spherical Projections and Data Analysis 1*" form the introductory material that introduces students to basic concepts in structural geology data measurement and analysis, including spherical projections. These concepts are revisited throughout the semester, including laboratories on fault stress, footwall diagrams and rotations.

Most of the students are simultaneously enrolled in my Field Methods course, which meets each Friday afternoon. SUNY New Paltz is conveniently close to numerous outcrops with lovely structures, many within a 20 minutes drive. The students will immediately begin applying these concepts while they learn compass use and other skills. During the semester they will collect bedding-cleavage measurements, collect measurements around outcrop scale and regional folds, un-tilt an angular unconformity, decipher fault relationships, and prepare a final geologic map with cross-sections.

Most of the students also accompany me on a four day trip to Acadia National Park in Maine, where there is ample opportunity to discuss these and other topics, as stress and strain, on the outcrop.

Later in the semester, after they have had time to master hand-drafted projections, I give then the laboratory "*Spherical Projections and Data Analysis 2*" on the orientation analysis program Orient, where we cover rotations, eigenvectors, contouring, eigenvalue diagrams, and related topics.

Any use of this material should be acknowledged. References are below for automatic contouring (Vollmer, 1995), and the Orient program (Vollmer, 2011). Reference to the contained material should be similar to:

Vollmer, F.W., 2012. Spherical Projections and Data Analysis. Teaching Structural Geology, Geophysics, and Tectonics in the 21st Century. On the Cutting Edge (https://serc.carleton.edu/NAGTWorkshops/structure/SGT2012/).

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Spherical Projections

Goals and Objectives

A primary goal of the course, and particularly of the laboratory, is to learn to visualize geometric problems in three dimensions. This is challenging, but the process can be rewarding, especially so if you approach each problem as a puzzle to be solved. Spherical projections are tools that allow solving three-dimensional problems, however what is more important is to learn to visualize the problem. Visualization leads to clarity and solutions, rote memorization leads to confusion and errors. The first laboratory will be an introduction to the use of spherical projections: orientation data measurements (strike, dip, trend, plunge, *etc.*), plotting of lines, planes and their normals, determining angles between planes, and data visualization. It is critical that you learn these skills now, as they will be used in subsequent labs, in the lectures, and in the field. Later, after you have sharpened your conceptual skills, you will learn the use of Orient, a computer program for analyzing orientation data. These concepts are further reinforced in Field Methods by collecting field data from folds and other structures, and analyzing it using the procedures you learn in the laboratory. Finally, we will discuss terminology related to spherical projections and the importance of correct notation, with the goal of improving scientific communication skills.

Concepts of Spherical Projections

A *spherical projection* is a mathematical transformation that maps points on the surface of a sphere to points on another surface, commonly a plane. Astronomers, cartographers, geologists, and others have devised numerous such projections over thousands of years, however two, the *stereographic projection* and the *equal-area projection*, are particularly useful in geology for displaying the angular relationships among lines and planes in three-dimensional space. A third projection, the *orthographic projection*, is less commonly used, but it is described here as it's properties are easily visualized. These are *azimuthal*, and are projections of a sphere onto a plane that preserve the directions (*azimuths*) of lines passing through the center of the projection. This is an important characteristic as azimuths, or horizontal angles from north (strike, trend, etc.), are a standard measurement in structural geology.

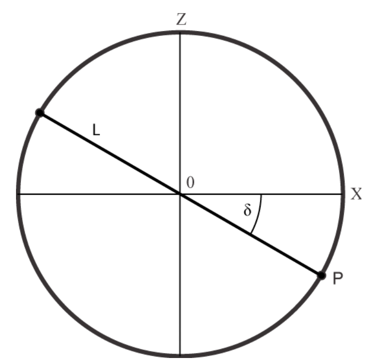

Figure 1. Definition of the point, P, on the unit sphere that defines the orientation of the undirected line L. The line is trending toward X (east) and it's plunge is δ. The Y coordinate axis (north) is into the page.

The orientations of lines and planes in space are fundamental measurements in structural geology. Since planes can be uniquely defined by the orientation of the plane's *pole*, or *normal*, it is sufficient to describe the orientation of a line. If only the *orientation* of a line, and not it's position, is being considered, then it can be described in reference to a unit sphere, of radius, $R = 1$. A right-handed cartesian coordinate system is defined with zero at the center of the sphere. A standard convention, used here, is to select X = east, Y = north, and Z = up (a common alternative is X = north, Y = east, and Z = down). A line, L, passing through the center of the sphere, the origin, will pierce the sphere at two diametrically opposed points (Figure 1).

If the line represents undirected *axial data* (as opposed to directed *vectorial data*), such as a fold axis or the pole to a joint plane, it is allowable to choose either point. In structural geology the convention is to choose the point on the lower hemisphere, P (the opposite convention is used in mineralogy). The three coordinates of point P are then known as *direction cosines*, and uniquely define the orientation of the line. More commonly, the *trend* (*azimuth* or *declination*) and *plunge* (*inclination*) of the line are given. In Figure 1, the trend of the line is 090°, and it's plunge is δ. It is a helpful reminder to always designate horizontal angles using three digits, where 000° = north, 090° = east, 180° = south, *etc.*, and to specify vertical angles using two digits.

An important tool for plotting lines and planes as data, and for geometric problem solving, is a *spherical net*. A spherical net is a grid formed by the projection of great and small circles, equivalent to lines of longitude and latitude. Nets are commonly either *meridianal* or *polar*, that is, projected onto a meridian (often the equator) or a pole. The terms *equator* and *pole* (or *axis*) will be used to refer to the equivalent geometric features on the net, it is essential to remember that they do not have an absolute reference frame, that is, the net axis is *not* equivalent to geographic north. In practice, as will be described in the laboratory, an overlay with an absolute geographic reference frame (north, east, south, etc.) is prepared.

The projections described here are *spherical projections*, so *equal-area projection* is assumed to mean *equal-area spherical projection.* In a later laboratory we will examine *hyperboloidal projections*, including equal-area and stereographic hyperboloidal projections. In these projections the surface of a *hyperboloid* is projected onto a plane. These are used in the context of strain analysis, and are unlikely to be confused with the more common spherical projections. All nets and data projections are prepared using Orient 2.1.1 (Vollmer, 2011).

Orthographic Spherical Projection

Orthographic projections are an important family of projections in which points are projected along parallel rays, as if illuminated by an infinitely distant light source. Figure 2 gives the geometric definition of the orthographic spherical projection. A corresponding orthographic polar net is shown in Figure 3, and an orthographic meridianal net is shown in

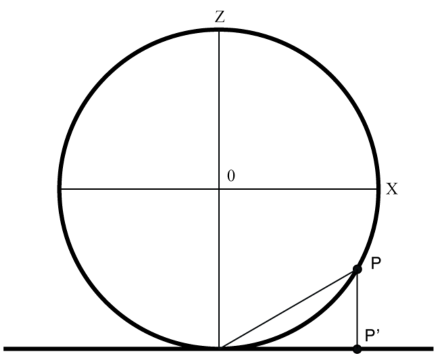

Figure 2. Geometric definition of the orthographic spherical projection.

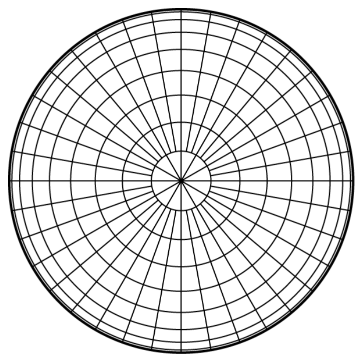

Figure 3. Polar orthographic net.

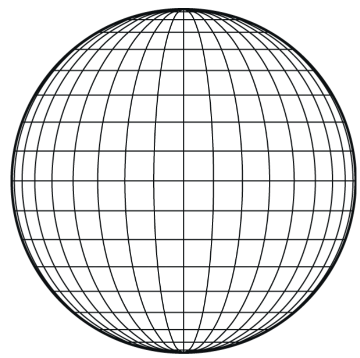

Figure 4. Meridianal orthographic net.

Figure 4. The projection of point P in the sphere to point P' on the plane is parallel to the cartesian axis Z, effectively giving a projection following a ray from Z equals positive infinity. This type of projection gives a realistic *perspective* view of a distant sphere, such as the earth viewed from space. It is azimuthal, but angles and area are not generally preserved. When plotting geologic data it is important that area, and therefore data densities, are preserved, so the orthographic projection unsuitable for such purposes. The net does, however, have other uses, such as the construction of block diagrams (e.g., Ragan, 2009).

Stereographic Spherical Projection

The *stereographic* or *equal-area* spherical projection is widely used in mineralogy and structural geology. It is defined geometrically by a ray passing from a point on the sphere (here Z = 1) through a point P on the sphere to the projected point

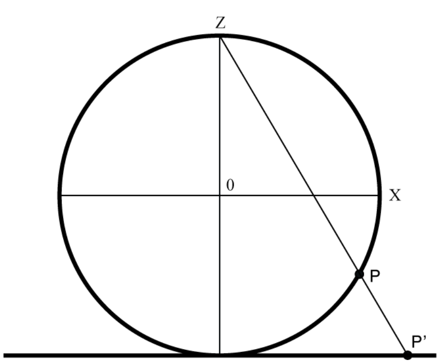

Figure 5. Geometric definition of the stereographic projection.

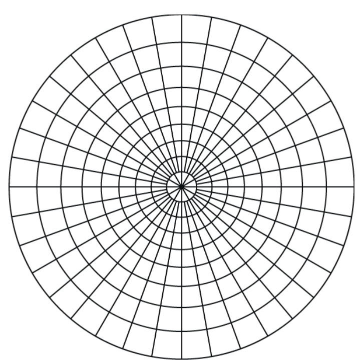

Figure 6. Polar stereographic net.

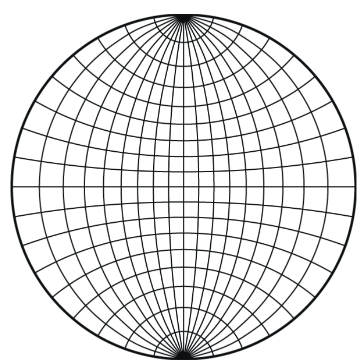

Figure 7. Meridianal stereographic net, also known as a stereonet or Wulff net.

P' on the plane (Figure 5). Note that all points on the sphere can be projected except the point of projection itself, which plots at infinity. The corresponding stereographic nets (Figures 6 and 7), however, plot only one hemisphere. Both hemispheres can be represented on the net, however the convention in structural geology is to use the *lower hemisphere*.

The meridinal stereographic net is known as a *stereonet*, or *Wulff net*, named after the crystallographer G.V. Wulff who published the first stereographic net in 1902 (Whitten, 1966). The stereonet is commonly used in mineralogy, however, the convention is to use the *upper hemisphere*. It is therefore good practice to clearly label all projections, for example "*lower-hemisphere stereographic projection.*"

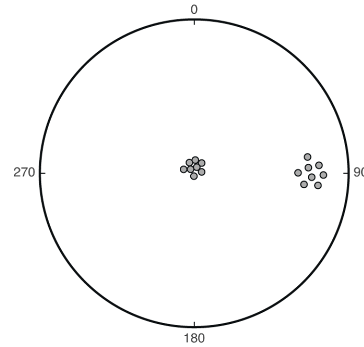

Figure 8. Lower-hemisphere stereographic projection of two data clusters showing density distortion. See text for discussion.

The projection is *azimuthal*, so lines passing through the center of the projection have true direction, these represent great circles. Note that area in Figure 7 is clearly distorted, the projection preserves angles (is *conformal*), but it does not preserve area. An important consequence is that great circles (such as meridians) and small circles project as circular arcs. These properties make it useful for numerous geometric constructions in structural geology (Bucher, 1944; Phillips, 1954; Badgley, 1959; Lisle and Leyshorn, 2004; Ragan, 2009).

The distortion of area, however, makes the stereographic projection less useful for studying rock fabrics, such as multiple orientations of bedding, joints, and crystallographic fabrics. Plotting such data is a descriptive statistical procedure intended to identify significant clusters, girdles, and other patterns. Figure 8 is a lower-hemisphere projection of two data clusters which are identical except for rotation. They have identical densities on the sphere, but this is distorted on the stereographic projection. An equal-area projection should be used instead (Sander, 1948, 1950, translated 1970; Phillips, 1954; Badgley, 1959; Turner and Weiss, 1963; Whitten, 1966; Fisher et al., 1987; Ragan, 2009).

Equal-Area Spherical Projection

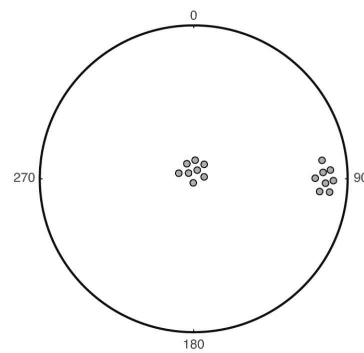

Figure 9. Lower hemisphere equal-area projection of two data clusters showing lack of density distortion.

The Lambert azimuthal equal-area spherical projection is arguably more widely used in structural geology than is the stereographic projection. It is not conformal, however an important characteristic is that it *preserves area*, so densities are not distorted (Figure 9). As discussed in the previous section, this makes it useful for the examination of rock fabrics, including the orientations of bedding, joints, and crystallographic fabrics (Billings, 1942; Sander, 1948, 1950, translated 1970; Phillips, 1954; Badgley, 1959; Turner and Weiss, 1963; Whitten, 1966; Fisher et al., 1987; Ragan, 2009). It appears widely in the geologic literature, and is the most likely of these projections to be encountered in scientific literature related to structural geology. Figures 10, 11 and 12 illustrate the geometric definition, polar net, and meridianal net respectively.

The term *azimuthal* indicates that, like stereographic and orthographic projections, lines passing through the center have true direction, and that it is projected onto a plane. This distinguishes it from other equal-area projections, which include the projection of a sphere onto conical and other surfaces, however, in structural geology, it can usually be referred to simply as an *equal-area projection* without ambiguity. The projection is also known as the Schmidt projection, after W. Schmidt who first used it in structural geology in 1925 (Turner

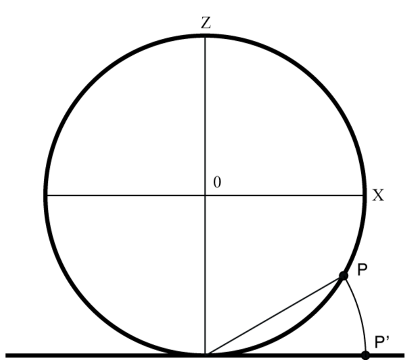

Figure 10. Geometric definition of the equal-area projection.

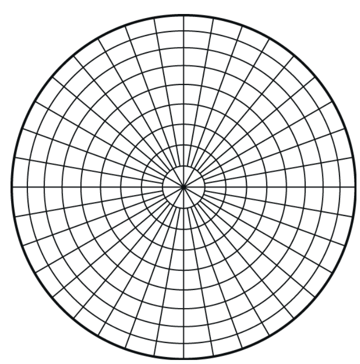
Figure 11. Polar equal-area net.

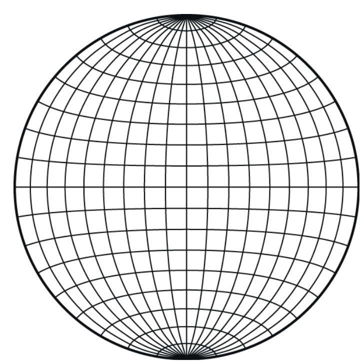
Figure 12. Meridianal equal-area net, or Schmidt net.

and Weiss, 1963), and the meridianal equal-area net, is known as a *Schmidt net* (Sander, 1948, 1950, translated 1970).

While the equal-angle properties of the stereographic projection make it useful for certain constructions, the important basic procedures that are commonly required are identical on the two projections. In this course we use only the equal-area projection and Schmidt nets for problem solving and data analysis.

Contouring and Eigenvectors

For most of the semester the Schmidt net will be used for plotting and analyzing data, this is the best way to visualize the geometry required to comprehend three-dimensional structures, a major objective of the course. Later in the semester, however, the use of the software program Orient (Vollmer, 2011) will be taught. Orient has many functions for plotting and analyzing orientation data, and is very useful for analyzing orientation data and preparing diagrams for publication (the spherical nets and plots presented here were prepared in Orient). Two important statistical procedures are *contouring* and calculating data *eigenvectors*.

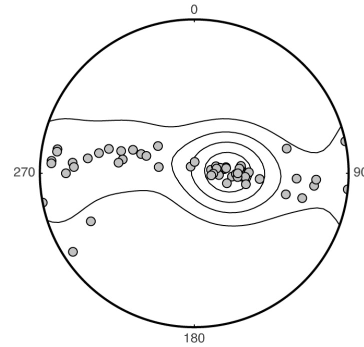

Figure 13. Contoured lower-hemisphere equal-area projection of 56 poles to bedding.

When analyzing orientation data a useful procedure is to contour the data to examine it for patterns such as clusters and girdles (Fisher, et al., 1987; Vollmer, 1995). A critical point in this procedure is that density calculations must be done on the sphere, prior to projection. Figure 13 is contoured plot of poles to bedding from an outcrop of folded graywackes in Albany County, New York (from Vollmer, 1981) which displays both cluster and girdle patterns. The relative strength of those can be computed, and plotted, using the computed eigenvectors.

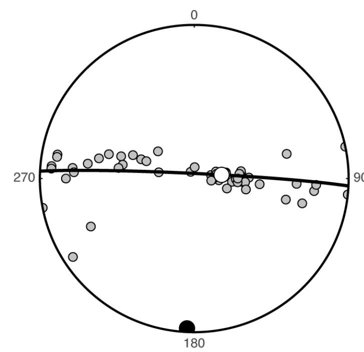

Figure 14. Lower hemisphere equal-area projection of 56 poles to bedding with maximum and minimum eigenvectors.

The concept of an *average* is familiar when dealing with *scalar* values like temperature. Determining an “average” or “best” value for orientation data is more complex (averaging trends and plunges separately does not work). In this course we will work with *vector* and *tensor* values (scalars and vectors are actually simple tensors). *Eigenvectors* are an important concept that allows the determination of the “best” values for a tensor, such as *principal stresses*. In the context of orientation data, imagine that each line plotted in Figure 14 is represented by a small mass at each of the two points where it pieces the sphere (see Figure 1). If you were to spin the sphere, it would have a natural tendency to spin about the axis of minimum density, this is the minimum eigenvector (the black point in Figure 14). If you were to roll the sphere, it would have a natural tendency to stop with the maximum density at the bottom, this is the maximum eigenvector (the white point in Figure 14). These two vectors are exactly 90° apart, and 90° from the intermediate eigenvector (not shown).

Terminology

The terminology of projections can be confusing, but it is important to use correct terms for effective scientific communication. The terms *stereographic projection* and *stereonet*, in particular, are frequently misused. Early references (Sander, 1948, 1950, translated 1970; Phillips, 1954; Badgley, 1959; Turner and Weiss, 1963; Whitten, 1966) are careful to use correct terminology, as are most current structural geology texts. Note that:

- The equal-area projection is not the stereographic projection
- The equal-area projection is not a type of stereographic projection
- A stereonet is a meridianal stereographic net, also known as a Wulff net (Badgley, 1959)
- A Schmidt net is a meridianal equal-area net (Sander, 1948, 1950, translated 1970)
- An equal-area net is not a stereonet
- A Schmidt net is not a stereonet
- A projection of data (e.g., Figure 8 or 9) is not a stereonet (or a net at all)
- The phrase “*equal-area stereographic projection”* is a contradiction

An additional term that is used in the context of spherical projections is s*tereogram*, which is a planar representation of a three-dimensional structure. It is used to refer to diagrams produced by stereographic projection, but may include block diagrams as well (Phillips, 1954). The phrase *equal-area stereogram* refers to a diagram produced by equal-area projection (Lisle and Leyshorn, 2004). Note that, in this usage, a stereographic net is a stereogram, and an equal-area net is an equal-area stereogram.

It is common to see data projections (similar to Figures 8 and 9) labeled “s*tereonet of poles to bedding*” or similar. Such a statement is incorrect and ambiguous. It is not a stereonet, and chances are good that it is an equal-area projection. Mislabeling equal-area projections as stereographic projections is common. Some books discuss the equal-area projection in a chapter titled “stereographic projection,” don't let this mislead you.

References Cited

Badgley, P.C., 1959. Structural methods for the exploration geologist. Harper and Brothers, New York, 280 p.

Billings, M.P., 1942. Structural geology. Prentice-Hall, New York, 473 pp.

Bucher, W.H., 1944. The stereographic projection, a handy tool for the practical geologist. Journal of Geology, v. 52, n.3, p. 191-212.

Fisher, N.I., Lewis, T., and Embleton, B.J., 1987. Statistical analysis of spherical data. Cambridge University Press, Cambridge, 329 pp.

Lisle, R.J. and Leyshorn, P.R., 2004. Stereographic projection techniques for geologists and civil engineers, 2nd edition. Cambridge University Press, Cambridge, 112 pp.

Phillips, F.C., 1954. The use of stereographic projection in structural geology. Edward Arnold, London, 86 p.

Ragan, 2009. Structural geology: An introduction to geometrical techniques, 4th edition. Cambridge University Press, Cambridge. 602 p.

Sander, B., 1970. An introduction to the study of fabrics of geological bodies, 1st English edition. Pergamon Press, Oxford. 641 p. [Translated from Sander, 1948, 1950. German edition. Springer-Verlag].

Turner, F.J. and Weiss, L.E., 1963. Structural analysis of metamorphic tectonites. McGraw-Hill Book Company, New York, 545 pp.

Vollmer, F.W., 1981. Structural studies of the Ordovician flysch and melange in Albany County, New York: M.S. Thesis, State University of New York at Albany, Advisor W.D. Means, 151 pp.

Vollmer, F.W., 1995. C program for automatic contouring of spherical orientation data using a modified Kamb method: Computers & Geosciences, v. 21, p. 31-49.

Vollmer, F.W., 2011. Orient 2.1.1: Spherical orientation data plotting program. www.frederickvollmer.com/orient.

Whitten, E.H.T., 1966. Structural geology of folded rocks. Rand McNally, Chicago, 663 pp.

Additional Sources

Davis, G.H., Reynolds, S.J., and Kluth, C.F., 2012. Structural geology of rocks and regions, 3rd edition. John Wiley, 839 pp.

Fosson, H., 2010. Structural geology. Cambridge University Press, Cambridge, 463 pp.

Hobbs, B.E., Means, W.D., and Williams, P.F., 1976. An outline of structural geology. Wiley, New York, 571 pp.

Phillips, F.C., 1971. The use of stereographic projection in structural geology, 3rd edition. Edward Arnold, London, 83 pp.

Pollard, D.D. and Fletcher, R.C., 2005. Fundamentals of structural geology. Cambridge University Press, Cambridge, 463 500 pp.

Twiss, R.J. and Moores, 2007. Structural geology, 2nd edition. W.H. Freeman, New York, 736 pp.

Van der Pluijm, B.A. and Marshak, S., 2004. Earth structure, 2nd edition. W.W. Norton, New York, 656 pp.

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GLG 338 Structure and Tectonics Dr. F.W. Vollmer

Spherical Projections and Data Analysis 1

This laboratory introduces basic concepts in structural geology data measurement and analysis. It forms the foundation of much of the subsequent laboratory work. Prior to the lab you should have read the handout on Spherical Projections, and Appendix B in the textbook. The problems to be solved and handed in are limited as we will spend a good portion of the lab working through concepts, and working together on examples.

Topics to be mastered include: measurements of lines, measurements of planes, the right-hand rule, spherical projections, the Schmidt net, plotting lines, plotting planes, the normal to a plane, the intersection of planes, the angle between planes, S (β) diagrams, and S-pole (π) diagrams.

For the following problems, hand in this answer sheet with *three* projections (for questions 1, 2a and 2b). Make sure your answers are in proper notation (azimuth notation and the right hand rule), and that the projections are neat and clearly labeled. Report preparation is one of our goals, so clarity, organization and neatness do count.

1. The following strike and dip measurements of bedding (S0) and cleavage (S1) were taken at three outcrops using the right-hand (right-dip) rule. Since cleavage is typically parallel to fold *axial planes*, bedding-cleavage intersections are generally parallel to *fold axes*. We will cover these concepts in depth later, but introduce them as examples of analyzing field data, similar to what will be collected in Field Methods.

Station	*Bedding* (S0)	*Cleavage* (S1)	*Intersection*	*Angle*
N1	050 - 28	012 - 70	____________	____________
N2	156 - 40	026 - 64	____________	____________
N3	038 - 74	194 - 82	____________	____________

   a) Determine the attitudes of the bedding-cleavage intersections in each outcrop by plotting the plane pairs. Note that bedding and cleavage are planes, so their intersection is a line whose orientation is given by *trend* and *plunge*. This line is a good predictor of the fold *axis*.

   b) Determine the *acute* angle between bedding and cleavage in each outcrop by plotting the poles to the planes. Do this on the same projection as question 1a.

   c) What can you say about the likely orientation of a fold in the area? (answer on back)

   d) Which outcrop is likely to be closest to the fold axial plane? Why? (answer on back)

2. The following measurements of bedding were taken around a fold:

051-81	051-69	351-30	339-40	247-68
044-74	040-58	007-41	244-84	056-87
321-30	308-42	254-64	236-86	336-26

   a) Construct a beta (S) diagram of bedding planes and determine the orientation of the fold axis as the location of maximum intersection density. ____________

   b) On a separate sheet construct a π (pi or S-pole) diagram of poles to bedding. Find the best fit great circle through the data points, and determine the fold axis as the pole to this great circle. ____________

   c) Comment on the relative merits of these two different methods. One method was widely used in the past, but rarely is today, why? (answer on back)

   d) What can you say about the possible orientation of the fold *axial plane* (not fold *axis*). (answer on back)

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Spherical Projections and Data Analysis 2

In this laboratory you will learn to use the spherical projection program Orient for advanced data analysis. Prior to the lab you should reread the handout on Spherical Projections, and Appendix B in the textbook. We will be talking about some of the more conceptually difficult concepts in more depth. The problems to be solved and handed in are limited as we will spend a good portion of the lab working through concepts, and working together on examples.

Topics to be mastered include: the difference between axial and vectorial data; properties of orthographic, equal-area, and stereographic projections; rotational transformations of data versus reference frames; eigenvectors and PGR diagrams; data contouring methods, including Kamb's method; diagram export in bitmap and vector graphics format, and import into Adobe Illustrator.

You will be downloading and installing Orient directly from my website, for which I will give instructions. Note that you have several options to print the diagram, the simplest is to import them into a Microsoft Word or LibreOffice document as a png. This will also allow you to type your laboratory neatly in report format. For posters and professional publication you will likely want to export svg vector graphics files and import them into Adobe Illustrator documents using the “Place” command.

The following problems are to be done using the Orient spherical projection program. Follow directions given in class to install and start Orient, and enter the following measurements of bedding taken around a fold:

051-81	051-69	351-30	339-40	247-68
044-74	040-58	007-41	244-84	056-87
321-30	308-42	254-64	236-86	336-26

1. Examine the data as poles to bedding, in orthographic, stereographic, and equal-area projections. Comment on the differences (use *equal-area projections* for the remaining questions). (answer on back)
2. Plot and print a S (β) diagram (only arcs, no poles) of bedding planes.
3. Plot and print a S-pole (π) diagram (only poles, no arcs) of poles to bedding with the eigenvector best fit great circle, and the eigenvector fold axis as the pole to this great circle. Hoe does this compare to the S diagram? (answer on back)
4. Plot a contoured diagram of the data using contour line and gradient options. What does contouring bring out? How does it help answer question 2d on the first lab? (answer on back)
5. Open the Data Statistics window and locate the undirected eigenvector solution. Note that the best fit axes are given in several ways, including direction cosines and trend and plunge. Give the trend and plunge of the best fit fold axis as determined by the *minimum eigenvector* of the orientation matrix, and compare this to the answers you got in Laboratory 1. (answer on back)
6. Plot a PGR (Point-Girdle-Random) graph for this data, and record the PGR indexes.
7. Open the provided data set of ice c-axis data from Kamb, 1959. Rotate the reference frame so the *maximum* eigenvector is vertical (to Z), and plot a grayscale contoured diagram using the Kamb method (without gradient) showing the data points. Print the diagram and describe the patterns that are brought out by the contouring. (answer on back)
8. Plot a PGR diagram for Kamb's data set, and record the PGR indexes. Compare these to Question 6, and explain the relevance of these. (answer on back)