

Calculation of Stream Discharge Required to Move Bed Material

Objective: Students will map two sections of a stream and calculate the depth, velocity, and discharge of flows required to move the stream bed load. Students will produce two cross-sectional profiles of the stream, one through a pool and one through a riffle section, and one longitudinal profile. Measurements of sediment size, depth, and velocity, will be determined at 1-meter intervals across each cross transect. Students will also calculate the approximate cross-sectional areas across the pool and riffle sections and associated discharges. Students will also determine the stream gradient along the longitudinal profile. From these data, students will employ hydrodynamic equations to calculate the critical shear stress, and mean flow velocity required to move the bed load. Students can then calculate what the discharge and stream width would be at the time of sediment movement, and compare these data with those calculated for "normal-flow" periods.

Part I – Field Procedures

A. Cross Sections

The cross section of a channel influences the stream flow velocity.

Use the 1.5 m range poles to collect **change of elevation data** at regular distance intervals (e.g., every 1 m). You do not have to pick locations at a fixed interval, but you do need to accurately represent the topography, so make sure to choose locations that are associated with topographic breaks, obstructions, and the **thalweg** (i.e., the lowest point of channel). Note the location and elevation of banks.

Each survey must extend across the entire channel and up onto the adjacent floodplain for several meters.

1. Two teams will collect measurements along different transects across the stream channel. One transect will cross a **“pool”** section and the other transect will cross a **“riffle”** section.
2. For every pole location that is in the water, be sure to measure the **depth** (d) of the water.
3. At every interval in the stream, measure the intermediate-axis **grain size** (D), using a meter stick, of the cobble that you randomly select at your feet. If the grains are small, estimate the size (e.g., gravel, sand, silt, mud). See Figure 1 for an example of how to organize a transect into discrete compartments.

Because fluvial deposits normally cover a wide range of grain sizes, it can be a challenge in the field to accurately characterize the average grain size. Furthermore, there are “patches” of bed material with distinct size distributions. Therefore, we will use pebble counts (Wolman 1954) to characterize grain size. At each location, reach down to the streambed with a single finger and pick up the first grain you touch. Do not look at where you are reaching. Measure the intermediate diameter of the grain (image it as an ellipsoid with three axes). Work systematically sampling across the entire width of the channel. Replace grains (especially large ones) either upstream or downstream where you won’t re-sample them.

4. Measure the stream **velocity** (U) at different positions along the transect. Select locations where the velocity is obviously different, if possible.

Current velocities should be measured upstream of where you are standing. Make sure no one is working immediately upstream or downstream while you are making current measurements. Orient the meter to the direction of flow. Current velocity varies vertically from 0 at the bed to a maximum value near the surface. The mean velocity can be estimated by averaging the flow velocities at just under the water surface and just above the streambed.

B. Longitudinal Profile

A longitudinal profile will allow the calculation of the stream **gradient** (S). It is typically surveyed along the main path of the current which ideally corresponds to the channel **thalweg**. If possible, select a transect that includes both pools and riffles, and is approximately 5–7 channel widths long. See Figure 2 for information on how to survey using the auto level. Make sure to capture changes in both topography and water depth. Record where each cross-section crosses the long profile.

Part II – Data Representation

Construct three graphs, two cross sections (riffle and pool sections), and one longitudinal section either using graph paper or a graphing program such as Excel. On the two cross sections, indicate **grain size**, **water depth**, and **mean velocity** measurements for each subsection (see Figure 1 for an example).

Construct a longitudinal profile using the surveying techniques illustrated in Figure 2.

(**Note:** for each section, plot both the stream bed elevation and water elevation data. This will give you two lines on your graph. *You'll find that you have to manipulate the data and/or the graphs' y axes in Excel to correctly display the data.*)

Part III – Data Analysis

Create a table of the following results:

1. Mean **grain size** (D) for each cross section.
2. Mean **depth** (d) for each cross section.
3. Mean **velocity** (U) for each cross section.
4. **Water area** (A) for each cross section in square meters. Area can be estimated by adding all the (depth x width) measurements for each section along both cross sections. See Figure 1 for a representation of how to calculate water area.
5. **Discharge** (Q) for each cross section: $Q = U A$. Discharge is calculated by multiplying the mean flow velocity of each cross section by the total water area.
6. Calculate the **gradient** (S) of the stream channel from the longitudinal profile data. This can be done in Excel by plotting water depth data vs. distance. Use the "Add Trendline" option under the "Chart" menu to create a linear regression line through the data points to determine the slope of the points. Remember to select "Display equation on chart" in the options window.

Part IV – Determination of Critical Shear Stress (τ_o) and Mean Flow Velocity (U)

What is the mean water velocity (U) required to transport the mean grain size (D)?

Show all calculations and don't forget the units!

Procedure

1. The mean fluid velocity (U) (cm/s) needed to transport a grain of size (D) is calculated using the **von Kármán–Prandtl equation**, otherwise known as the *Law of the Wall* (for rough boundaries)

$$\frac{U}{\mu_*} = \left[\frac{1}{k} \ln \left(\frac{y}{y_o} \right) \right] + A \quad (1)$$

where:

μ_* = shear velocity (cm/s)

$\kappa = 0.4$ (von Kármán's constant) (dimensionless)

y = height (cm) above boundary surface where the mean flow velocity U is reached. Use height = $0.4 d$

y_o = height (cm) above stream bed where $U = 0$, usually determined graphically through experimentation, but can be estimated by: $y_o = D/30$

A = coefficient that varies with the *type* of surface roughness
(use 8.5 for well-sorted, close-packed grains)

Therefore, substituting the above values into equation (1) yields:

$$U = \left[2.5 \ln \left(\frac{y}{y_o} \right) + 8.5 \right] \mu_* \quad (2)$$

2. To solve equation (2) you first need to calculate the shear velocity (μ_*) where:

$$\mu_* = \sqrt{\frac{\tau_o}{\rho}} \quad (3)$$

3. To solve equation (3) you first need to calculate the boundary shear stress (τ_o) (g/cm s²) (shear force applied to a unit area of the sediment surface exerted by the fluid). This is done by using the Shields diagram (Figure 3).

a. Calculate the value of the quantity:

$$\frac{D}{\nu} \sqrt{0.1 \left(\frac{\rho_s}{\rho} - 1 \right) g D} \quad (4)$$

where:

$\nu = 0.01 \text{ cm}^2/\text{s}$ for water at 20 °C

$g = 981 \text{ cm/s}^2$ (gravitational acceleration)

$\rho_s = 2.65 \text{ g/cm}^3$ (sediment density)

$\rho = 1.00 \text{ g/cm}^3$ (water density)

D = mean grain size (in cm)

b. Find this value on the Shields diagram (Figure 3) and read downward, following the line, to the Shields curve. Then read over to the y-axis. This new value is the Shields critical boundary stress (θ_c) (dimensionless).

$$\theta_c = \frac{\tau_o}{(\rho_s - \rho) g D} \quad (5)$$

- c. Rearrange equation (5) and solve for τ_o .
4. Place the τ_o value into equation (3) to get μ_* .
5. Put the μ_* value into equation (2) to get (finally!) the mean flow velocity (U).

Part V – Stream Characteristics During Sediment Transport

What is the discharge and stream width at the time of sediment movement?

Procedure

1. Knowing τ_o and S , calculate the water depth (y) at the time of sediment movement, where:

$$\tau_o = \rho g y S \quad (6)$$

2. On one of your cross sections, draw a horizontal line that represents the water level at the time of sediment movement. Extend this line until it intersects with the land surface. Estimate the **width** of the stream when water depth = y .
3. Estimate the discharge (Q) associated with water depth y , where:

$$Q = U y \text{ width} \quad (7)$$

Part VI – Questions

- Q1.** Is the mean water velocity substantially different across the two transects? If so, what might cause this difference (i.e., can you relate velocity changes to other stream characteristic(s))?
- Q2.** Do variations in cross-sectional area relate to changes in mean velocity? If so, can you relate these variations to a geomorphic feature(s)?
- Q3.** Does the mean grain size substantially differ (qualitatively) between the two cross sections? If so, what might this be related to?
- Q4.** Does discharge differ substantially between transects? Why or why not?
- Q5.** Does the water surface parallel the stream bed surface in the longitudinal profile? If not, what could explain this variation?
- Q6.** Why would the estimated discharge in Part IV (#3) be lower than the probable value?

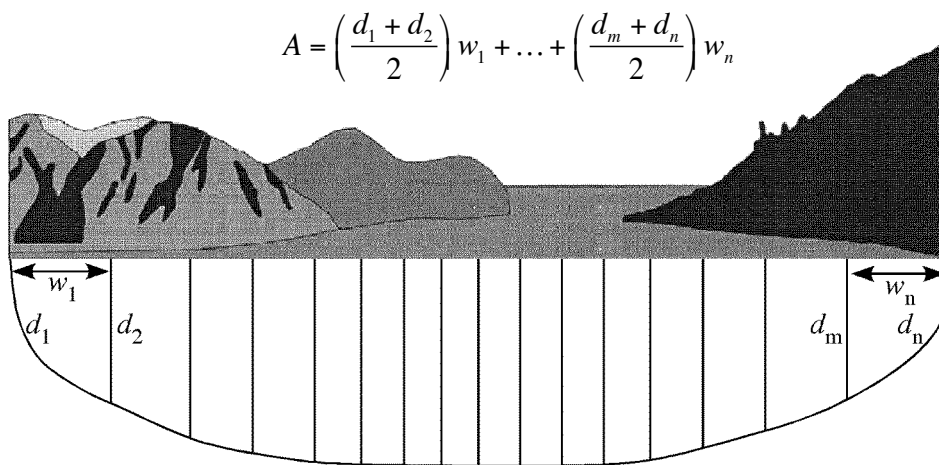


Figure 1. Stream cross section illustrating mid-section method to determine cross-sectional area (after Rantz et al., 1982). A = cross-sectional area, d = depth, w = width.

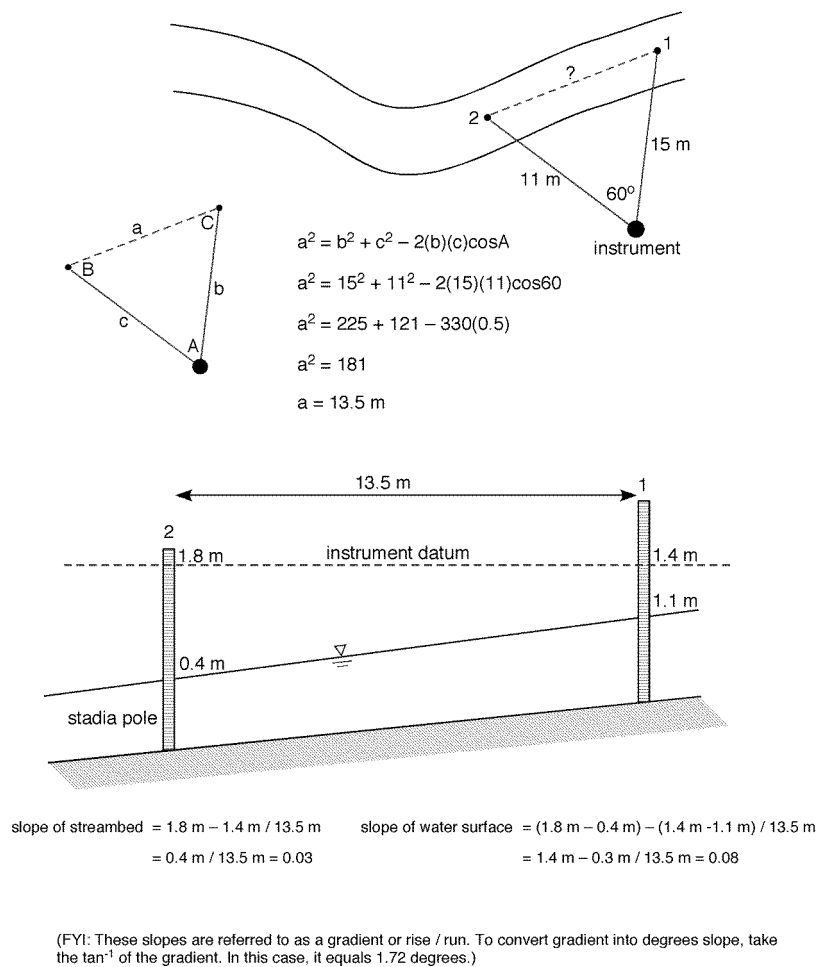


Figure 2. Stream surveying techniques.

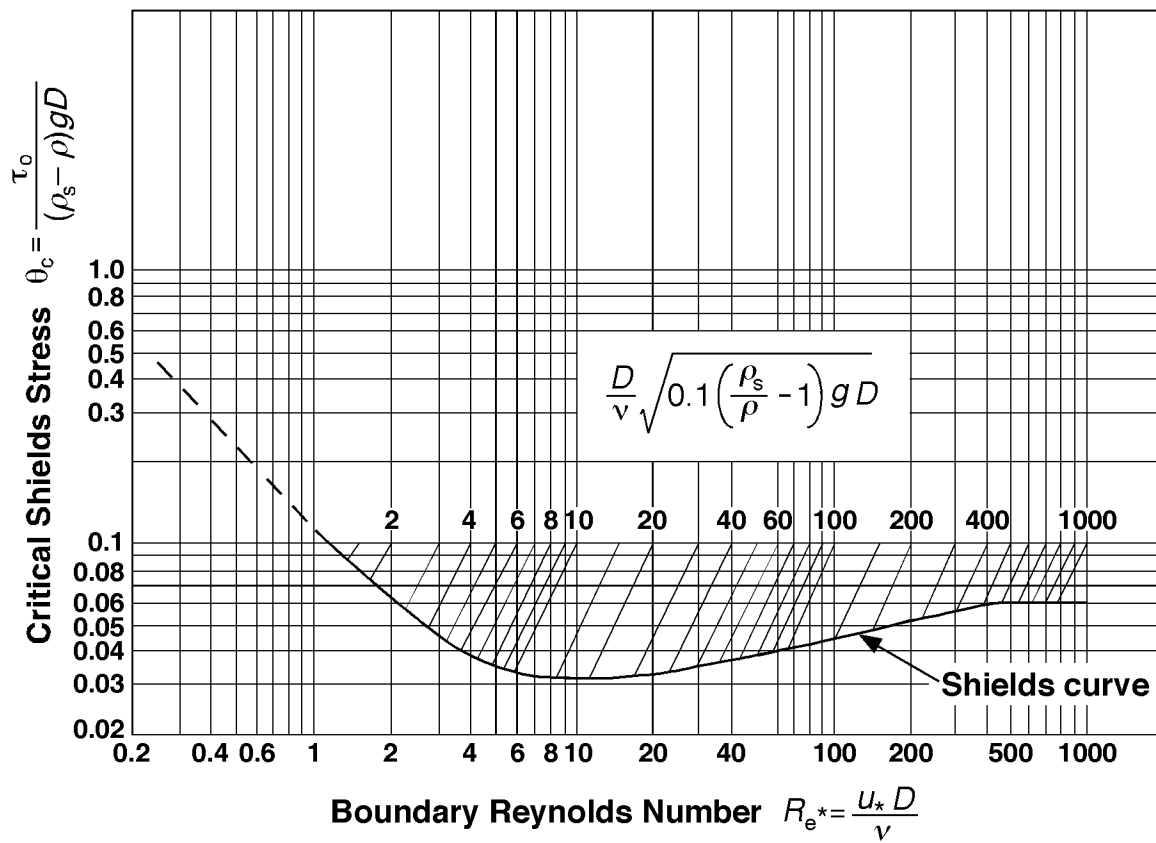


Figure 3. Shields diagram.