$\qquad$


Here are four phase diagrams: for 1-component, 2-component, 3-component, and 4-component systems.

Note that the diagram for the 1-component system has the stable minerals (or liquid) labeled in each field. The other diagrams have the reactions labeled (although the reactions are not balanced).

Abbreviations on phase diagrams

| Dsp | diaspore | $\mathrm{AlO}(\mathrm{OH})$ | Srp | serpentine | $\mathrm{Mg}_{3} \mathrm{Si}_{2} \mathrm{O}_{5}(\mathrm{OH})_{4}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Qz | quartz | $\mathrm{SiO}_{2}$ | Fo | forsterite | $\mathrm{Mg}_{2} \mathrm{SiO}_{4}$ |
| Py | pyrophyllite | $\mathrm{Al}_{2} \mathrm{Si}_{4} \mathrm{O}_{10}(\mathrm{OH})_{2}$ | Di | diopside | $\mathrm{CaMgSi}_{2} \mathrm{O}_{6}$ |
| Ka | kaolinite | $\mathrm{Al}_{2} \mathrm{Si}_{2} \mathrm{O}_{5}(\mathrm{OH})_{4}$ | Tr | tremolite | $\mathrm{Ca}_{2} \mathrm{Mg}_{5} \mathrm{Si}_{8} \mathrm{O}_{22}(\mathrm{OH})_{2}$ |
| Ky | kyanite | $\mathrm{Al}_{2} \mathrm{SiO}_{5}$ | Tc | talc | $\mathrm{Mg}_{3} \mathrm{Si}_{4} \mathrm{O}_{10}(\mathrm{OH})_{2}$ |
| Co | corundum | $\mathrm{Al}_{2} \mathrm{O}_{3}$ | En | enstatite | $\mathrm{Mg}_{2} \mathrm{Si}_{2} \mathrm{O}_{6}$ |
| Gr | grossular | $\mathrm{Ca}_{3} \mathrm{Al}_{2} \mathrm{Si}_{3} \mathrm{O}_{12}$ | Ant | anthophyllite | $\mathrm{Mg}_{7} \mathrm{Si}_{8} \mathrm{O}_{22}(\mathrm{OH})_{2}$ |

1. What is the maximum number of phases (minerals or melt or $\mathrm{H}_{2} \mathrm{O}$ involved in any reaction on each of the diagrams? Fill in the table below. Look carefully!

| Diagram | Max number of <br> phases in a <br> reaction |
| :--- | :--- |
| Diagram 1-1 component |  |
| Diagram 2-2 components |  |
| Diagram 3-3 components |  |
| Diagram 4-4 components |  |

What about chemical components? Components are the minimum number of ingredients needed to describe the compositions of the phases involved. We use the variable C to represent the minimum number of components.

In Diagram 1, all phases are the same composition. They are all $\mathrm{SiO}_{2}$. So there is only one component, and that component is silicon dioxide $\left(\mathrm{SiO}_{2}\right) . \mathrm{C}=1$.
2. In Diagram 3, we need more than one component to describe all compositions. We need 3. $\mathrm{C}=$ 3. Typically it works to use oxides as components*. What oxides (list 3) are needed to make up the compositions of all minerals in Diagram 3? List them.
3. For Diagram 4, we need 4 components. $C=4$. List the four oxides that can be used to make up the compositions of all minerals.

[^0]4. What about Diagram 2. The phases are only identified as A, B, C, and D. We need 2 components. And we can select from compositions equivalent to A, B, C, or D. Figure out which two you need.

If for example you think the answer is to use compositions equal to A and C , then the other two ( B and C ) must be equivalent to some combination of A and C . But this does not work. So, compositions equivalent to A and C are not acceptable choices for components. It is complicated but if you mess around you should be able to figure out 2 components that work.

What are the two correct components?

When two reactions intersect, it usually creates an invariant point at 1 specific temperature and 1 specific pressure. (There are, however, times when reactions cross without creating an invariant point - discussed briefly below.) At the conditions of the invariant point, all the phases that are involved in the intersecting reactions may coexist.

I have marked invariant points with circles on each of the three diagrams below.


Diagram 1 -
1 component system


Diagram 2 -
2 component system


Diagram 3 -
3 component system
5. What is the maximum number of phases (minerals or melt or $\mathrm{H}_{2} \mathrm{O}$ ) that can exist at the invariant points shown on each of the diagrams on the previous page? We use the variable P for this number. Look at the larger diagrams on the first page of this handout and fill in the table below.

| Point/Diagram | Max number of <br> phases at <br> reaction <br> intersections <br> (P) | list the phases that coexist at the invariant <br> point |
| :--- | :--- | :--- |
| I1 on Diagram 1-1 |  |  |
| I2 on Diagram 2-2 |  |  |
| I3 on Diagram 3-3 |  |  |

Invariant points occur at one specific P and T . So we say they have zero degrees of freedom. We use F as the variable for degrees of freedom. So, at an invariant point, $\mathrm{F}=0$. We cannot change P OR T and stay on the point.

On a phase diagram, reaction lines intersect at invariant points. There are occasionally exceptions, discussed below, but in general this is true. Along a reaction line, we can change either P or T independently and slide up and down the line. But, once we specify a change in T , there is only one change in P that we can make and still stay on the line. So, for reaction lines, F $=1$.

In open spaces, we can change both P and T independently (within limits) and still stay in the same field ( field $=$ a space between reaction lines). $\mathrm{So}, \mathrm{F}=2$.
6. So, using the numbers you put into the previous tables, complete this the one below. You will have to logic out values for the boxes marked with question marks. You can do this by looking at the number patterns. If you do all this correctly, you have derived the phase rule.

| C | P for $\mathrm{F}=2$ | P for $\mathrm{F}=1$ | P for $\mathrm{F}=0$ |
| :--- | :--- | :--- | :--- |
| \# components | (max) number of <br> phases stable in a <br> field | (max) number of <br> phases in a reaction | (max) number of <br> phases at an invariant <br> point |
| 1 |  |  |  |
| 2 | $?$ |  |  |
| 3 | $?$ |  | $?$ |
| 4 | $?$ |  |  |

Above, I asked you to count reactions around invariant points, ut I did not ask you to look at Diagram 4. This is because many of the reactions in Diagram 4 are degenerate. That means they contain fewer phases than are allowed by the phase rule. This means that an invariant point may also contain fewer than the maximum number of phases allowed by the phase rule.
7. Look at Diagram 4. List two reactions that are not degenerate (in Diagram 4).
8. List two reactions that are degenerate (in Diagram 4).

Another complication with Diagram 4 is that there are indifferent crossings where reactions cross but do not create an invariant point.

Look at the intersection in the upper right hand corner of Diagram 4. The two reactions there involve $\mathrm{Br}, \mathrm{Pe}, \mathrm{H}_{2} \mathrm{O}, \mathrm{Tr}$, En, Di, and Qz. This is 7 phases. Not allowed by the Phase Rule. So the intersection is NOT an invariant point.
9. Aren't you glad I did not ask many questions about Diagram 4 ?

## Phase Diagram Problem 0

The phase diagram below includes reactions for a system that includes 6 minerals (listed in the table). The numbers and letters on points, lines and spaces are usually not preseent but are included to help you give clear answers to the questions below.

| mineral | abbrev. | formula |
| :--- | :--- | :--- |
| anorthite | An | $\mathrm{CaAl}_{2} \mathrm{Si}_{2} \mathrm{O}_{8}$ |
| CaTs <br> (calcium <br> tschermaks <br> pyroxene) | CaTs | $\mathrm{CaAl}_{2} \mathrm{SiO}_{6}$ |
| grossular | Gr | $\mathrm{Ca}_{3} \mathrm{Al}_{2} \mathrm{Si}_{3} \mathrm{O}_{12}$ |
| quartz | Q | $\mathrm{SiO}_{2}$ |
| wollastonite | Wo | $\mathrm{CaSiO}_{3}$ |
| kyanite | Ky | $\mathrm{Al}_{2} \mathrm{SiO}_{5}$ |

1. List the oxides needed to make up any of the minerals.
2. Usually, the number of oxides is equal to the number of chemical components necessary to describe a system. So, how many components are in this system?
3. The phase rule says $\mathrm{C}+2=\mathrm{P}+\mathrm{F}$ where $\mathrm{C}=$ the number of components, $\mathrm{P}=$ the maximum number of phases that can coexist (in this example they are all minerals), and $\mathrm{F}=$ degrees of freedom ( 0 at an invariant point, 1 on a reaction line, and 2 in an open space). So, how many phases may coexist at an invarinat point, on a line, or in an open space for the system we are talking about? (Fill in the blanks.)


4. List all the possible minerals that may coexist at I1.
5. List all possible minerals that may coexist at I2.

I3 is NOT an invariant point - instead it is called an "indifferent crossing" - because there are too many minerals in the reactions that cross there. (What this means is that reaction 4 and reaction 5 do not apply to the same composition rocks.)

Some of the reactions on this diagram involve 4 minerals. Others involve only 3. Reactions that involve fewer than the maximum allowable number of minerals are "degenerate reactions." Degenerate reactions involve fewer chemical components than other reactions in a system.
7. List and balance all the degenerate reactions.
8. In this system, the maximum number of minerals that can coexist is 5. And only at invariant points. So, if you are asked where $\mathrm{CaTs}+\mathrm{Q}+\mathrm{Ky}+\mathrm{An}+\mathrm{Wo}+\mathrm{Gr}$ coexist, you know it is nowhere.

But, what about $\mathrm{CaTs}+\mathrm{Q}+\mathrm{Ky}+\mathrm{An}+\mathrm{Wo}$ ( 5 minerals). You need only check out the two invariant points to see if the assemblage can be. Where, if anywhere, can $\mathrm{CaTs}+\mathrm{Q}+\mathrm{Ky}+\mathrm{An}+\mathrm{Wo}+\mathrm{Gr}$ be stable together?

9. Where, if anywhere, can $\mathrm{CaTs}+\mathrm{Q}+\mathrm{Gr}+\mathrm{An}+$ Wo be stable together?
10. 4 minerals may coexist on a reaction line (and at invariant points). So, if you are asked where $\mathrm{CaTs}+\mathrm{Q}+\mathrm{Ky}+\mathrm{An}$ coexist, you need only look at reaction lines. Where, if anywhere, can these 4 minerals exist stably together?
11. Where, if anywhere, can $\mathrm{Gr}+\mathrm{CaTs}+\mathrm{Q}+\mathrm{Ky}$ be stable together?
12. 3 minerals may coexist in open spaces - perhaps more than one open space - between reaction lines (and on lines and at invariant points). So, if you are asked where CaTs+Gr+An coexist, you need only look at the spaces and the reactions that border them. Where, if anywhere, can these 3 minerals exist stably together?
13. Where, if anywhere, can $\mathrm{Wo}+\mathrm{Ky}+\mathrm{CaTs}$ be stable together? (This is a bit tricky.)
14. What about 2 minerals coexisting? Where on the diagram can $\mathrm{Gr}+\mathrm{Ky}$ coexist?
15. Where on the diagram can An+Wo coexist?
16. Where on the diagram can Gr+Wo coexist?
17. And, what about single minerals. Where on the diagram can An exist?
18. Where on the diagram can Wo exist?
19. Where on the diagram can Gr exist?

## Phase Diagram Problem

A schematic diagram involving 6 minerals is shown below. Reactions are numbered and open spaces are labeled with letters.


1. Consider the diagram shown. How many components are needed to describe the system that is involved?
2. Which minerals are stable everywhere on this diagram?
3. What about the other minerals. List them and describe where they are stable.

| mineral/assemblage | where stable? |
| :---: | :---: |
| Sil |  |
| $\mathrm{Sil}+\mathrm{Ge}$ |  |
| $\mathrm{Gr}+\mathrm{Sil}$ |  |
| Ky + Sil |  |
| $\mathrm{Ky}+\mathrm{Q}+\mathrm{Gr}$ |  |
| $\mathrm{Sil}+\mathrm{Gr}+\mathrm{Q}$ |  |
| $\mathrm{Ky}+\mathrm{Q}+\mathrm{An}$ |  |
| $\mathrm{Ky}+\mathrm{And}+\mathrm{Sil}+\mathrm{Gr}$ |  |
| $\mathrm{An}+\mathrm{Wo}+\mathrm{Gr}+\mathrm{Q}$ |  |
| $\mathrm{An}+\mathrm{Wo}+\mathrm{Gr}+\mathrm{And}$ |  |
| $\mathrm{An}+\mathrm{Ge}+\mathrm{Wo}+\mathrm{Gr}+\mathrm{Sil}$ |  |
| $\mathrm{Ky}+\mathrm{And}+\mathrm{Sil}+\mathrm{Gr}+\mathrm{Q}$ |  |
| $\mathrm{Wo}+\mathrm{Ky}+\mathrm{And}+\mathrm{Sil}+\mathrm{Gr}+\mathrm{Q}$ |  |

## Phase Diagram Problem 2

This diagram shows reactions involving diaspore (Dsp), kyanite (Ky), zoisite (Zo), quartz (Qz), margarite $(\mathrm{Mg})$, corundum $(\mathrm{Co})$, and $\mathrm{H}_{2} \mathrm{O}$. There are 4 oxide components: $\mathrm{CaO}, \mathrm{Al}_{2} \mathrm{O}_{3}, \mathrm{SiO}_{2}, \mathrm{H}_{2} \mathrm{O}$.



Mark your answers to these question on the diagrams, or say nowhere, or say everywhere.
5. Where on this diagram is margarite ( Mg ) stable?
6. Where on this diagram is anorthite (An) stable?

7. Where on the diagram is an assemblage containing margarite and anorthite (Mg + An) stable?
8. Where on the diagram is an assemblage containing margarite and quartz ( $\mathrm{Mg}+$ Qz) stable?

9. Where on the diagram is an assemblage containing diaspore, kyanite and $\mathrm{H}_{2} \mathrm{O}$
( $\mathrm{Dsp}+\mathrm{Ky}+\mathrm{H}_{2} \mathrm{O}$ ) stable?
10. Where on the diagram is an assemblage containing diaspore, anorthite, quartz and margarite ( $\mathrm{Dsp}+\mathrm{An}+\mathrm{Qz}+\mathrm{Mg}$ ) stable?

11. Where on the diagram is an assemblage containing diaspore, kyanite, margarite and $\mathrm{H}_{2} \mathrm{O}\left(\mathrm{Dsp}+\mathrm{Ky}+\mathrm{Mg}+\mathrm{H}_{2} \mathrm{O}\right)$ stable?
12. Where on the diagram is an assemblage containing diaspore, corundum and $\mathrm{H}_{2} \mathrm{O}$ ( $\mathrm{Dsp}+\mathrm{Co}+\mathrm{H}_{2} \mathrm{O}$ ) stable?

13. Where on the diagram is an assemblage containing diaspore, kyanite, zoisite, margarite and $\mathrm{H}_{2} \mathrm{O}$ ( $\mathrm{Dsp}+\mathrm{Ky}+\mathrm{Zo}+\mathrm{Mg}+\mathrm{H}_{2} \mathrm{O}$ ) stable?
14. Where on the diagram is an assemblage containing margarite + quartz + anorthite + zoisite $+\mathrm{H}_{2} \mathrm{O}(\mathrm{Mg}+\mathrm{Qz}+\mathrm{An}$ $\left.+\mathrm{Zo}+\mathrm{H}_{2} \mathrm{O}\right)$ stable?

15. Where on the diagram is an assemblage containing quartz, diaspore, kyanite, zoisite, margarite and $\mathrm{H}_{2} \mathrm{O}$
$\left(\mathrm{Qz}+\mathrm{Dsp}+\mathrm{Ky}+\mathrm{Zo}+\mathrm{Mg}+\mathrm{H}_{2} \mathrm{O}\right)$ stable?
16. Where on the diagram is an assemblage containing diaspore, quartz, anorthite, corundum and $\mathrm{H}_{2} \mathrm{O}$
(Dsp+Qz+An+Co+ $\mathrm{H}_{2} \mathrm{O}$ ) stable?

## Phase Diagram Problem 3

A schematic diagram involving 6 minerals is shown below. Reactions are numbered and open spaces are labeled with letters. Note that reaction 3 and reaction 6 are the same reaction on different sides of the invariant point. And reaction 1 and reaction 5 are the same reaction on different sides of the invariant point.
abbreviations:
dsp $=$ diaspore $=\mathrm{AlO}(\mathrm{OH})$
zo $=$ zoisite $=\mathrm{Ca}_{2} \mathrm{Al}_{3} \mathrm{Si}_{3} \mathrm{O}_{12}(\mathrm{OH})$
$\mathrm{ky}=$ kyanite $=\mathrm{Al}_{2} \mathrm{SiO}_{5}$
$\mathrm{Mg}=$ margarite $=\mathrm{CaAl}_{2}\left(\mathrm{Al}_{2} \mathrm{Si}_{2}\right) \mathrm{O}_{10}(\mathrm{OH})_{2}$
$\mathrm{H} 2 \mathrm{O}=$ water fluid

7. What is the maximum number of phases that may coexist anywhere on this diagram?
8. What is the maximum number of phases that may coexist stably in an open space on this diagram?
9. Bonus question. Two of the reactions pass through the invariant point and out the other side. The other reactions do not. Explain this.


T
Fill in the table below to show where each of the minerals or mineral assemblages are stable.

| mineral/assemblage | where stable? |
| :--- | :--- |
| Zo |  |
| Mg |  |
| $\mathrm{Mg}+\mathrm{Zo}$ |  |
| Dsp +Zo |  |
| $\mathrm{Mg}+\mathrm{Py}+\mathrm{Ky}$ |  |
| $\mathrm{Mg}+\mathrm{Zo}+\mathrm{Py}$ |  |
| $\mathrm{Ky}+\mathrm{Py}+\mathrm{Dsp}+\mathrm{H}_{2} \mathrm{O}$ |  |
| $\mathrm{Py}+\mathrm{Zo}+\mathrm{Ky}+\mathrm{Dsp}$ |  |
| $\mathrm{Mg}+\mathrm{Ky}+\mathrm{Zo}+\mathrm{Dsp}$ |  |
| $\mathrm{Py}+\mathrm{Dsp}+\mathrm{Ky}+\mathrm{Mg}+\mathrm{H}_{2} \mathrm{O}$ |  |
| $\mathrm{Dsp}+\mathrm{Mg}+\mathrm{Zo}+\mathrm{Ky}+\mathrm{Py}$ |  |
| $\mathrm{Zo}+\mathrm{Ky}+\mathrm{Py}+\mathrm{Mg}+\mathrm{H}_{2} \mathrm{O}$ |  |
| $\mathrm{Py}+\mathrm{Mg}+\mathrm{Zo}+\mathrm{Ky}+\mathrm{H}_{2} \mathrm{O}$ |  |
| $\mathrm{Dsp}+\mathrm{Py}+\mathrm{Mg}+\mathrm{Zo}+\mathrm{Ky}+\mathrm{H}_{2} \mathrm{O}$ |  |


[^0]:    *When will oxide components not work? It can be difficult to tell. But suppose we are talking about these minerals: wollastonite $\quad \mathrm{CaSiO}_{3}$
    $\mathrm{CaTs} \quad \mathrm{CaAl}_{2} \mathrm{SiO}_{6}$
    grossular $\quad \mathrm{Ca}_{3} \mathrm{Al}_{2} \mathrm{Si}_{3} \mathrm{O}_{12}$
    corundum $\quad \mathrm{Al}_{2} \mathrm{O}_{3}$
    The oxides are $\mathrm{CaO}, \mathrm{SiO}_{2}$, and $\mathrm{Al}_{2} \mathrm{O}_{3}$. That could be 3 components.
    But, all these compositions can be described as combinations of $\mathrm{Al}_{2} \mathrm{O}_{3}$ and $\mathrm{CaSiO}_{3}$, which is two components. Since it is the minimum number that counts, these four minerals involve 2 components only.

