

Here are four phase diagrams: for 1-component, 2-component, 3-component, and 4-component systems.

Note that the diagram for the 1-component system has the stable minerals (or liquid) labeled in each field. The other diagrams have the reactions labeled (although the reactions are not balanced).

Abbreviations on phase diagrams

Dsp	diaspore	AlO(OH)	Srp	serpentine	Mg ₃ Si ₂ O ₅ (OH) ₄
Qz	quartz	SiO ₂	Fo	forsterite	Mg_2SiO_4
Ру	pyrophyllite	$Al_2Si_4O_{10}(OH)_2$	Di	diopside	CaMgSi ₂ O ₆
Ka	kaolinite	$Al_2Si_2O_5(OH)_4$	Tr	tremolite	$Ca_2Mg_5Si_8O_{22}(OH)_2$
Ky	kyanite	Al_2SiO_5	Tc	talc	$Mg_3Si_4O_{10}(OH)_2$
Co	corundum	Al_2O_3	En	enstatite	$Mg_2Si_2O_6$
Gr	grossular	$Ca_3Al_2Si_3O_{12}$	Ant	anthophyllite	$Mg_7Si_8O_{22}(OH)_2$

1. What is the maximum number of phases (minerals or melt or H_2O involved in any reaction on each of the diagrams? Fill in the table below. Look carefully!

Diagram	Max number of phases in a reaction
Diagram 1 - 1 component	
Diagram 2 - 2 components	
Diagram 3 - 3 components	
Diagram 4 - 4 components	

What about *chemical components*? Components are the minimum number of ingredients needed to describe the compositions of the phases involved. We use the variable C to represent the minimum number of components.

In Diagram 1, all phases are the same composition. They are all SiO_2 . So there is only one component, and that component is silicon dioxide (SiO_2) . C = 1.

2. In Diagram 3, we need more than one component to describe all compositions. We need 3. C = 3. Typically it works to use oxides as components*. What oxides (list 3) are needed to make up the compositions of all minerals in Diagram 3? List them.

3. For Diagram 4, we need 4 components. C = 4. List the four oxides that can be used to make up the compositions of all minerals.

*When will oxid	e components not work? It can be difficult to tell. But suppose we are talking about these minerals:
wollastonite	CaSiO ₃
CaTs	CaAl ₂ SiO ₆
grossular	$Ca_3AI_2Si_3O_{12}$
corundum	Al ₂ O ₃
The oxides are CaO, SiO ₂ , and Al ₂ O ₃ . That could be 3 components. But, all these compositions can be described as combinations of Al ₂ O ₃ and CaSiO ₃ , which is two components. Since it is the minimum number that counts, these four minerals involve 2 components only.	

4. What about Diagram 2. The phases are only identified as A, B, C, and D. We need 2 components. And we can select from compositions equivalent to A, B, C, or D. Figure out which two you need.

If for example you think the answer is to use compositions equal to A and C, then the other two (B and C) must be equivalent to some combination of A and C. But this does not work. So, compositions equivalent to A and C are not acceptable choices for components. It is complicated but if you mess around you should be able to figure out 2 components that work.

What are the two correct components?

When two reactions intersect, it usually creates an *invariant point* at 1 specific temperature and 1 specific pressure. (There are, however, times when reactions cross without creating an invariant point – discussed briefly below.) At the conditions of the invariant point, all the phases that are involved in the intersecting reactions may coexist.

I have marked invariant points with circles on each of the three diagrams below.

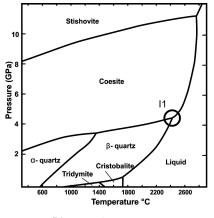


Diagram 1 -1 component system

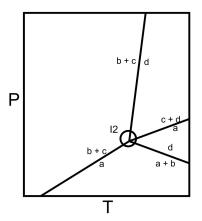


Diagram 2 -2 component system

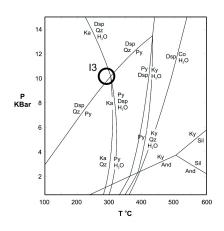


Diagram 3 -3 component system

5. What is the maximum number of phases (minerals or melt or H_2O) that can exist at the invariant points shown on each of the diagrams on the previous page? We use the variable P for this number. Look at the larger diagrams on the first page of this handout and fill in the table below.

Point/Diagram	Max number of phases at reaction intersections (P)	list the phases that coexist at the invariant point
I1 on Diagram 1 - 1		
I2 on Diagram 2 - 2		
I3 on Diagram 3 - 3		

Invariant points occur at one specific P and T. So we say they have zero *degrees of freedom*. We use F as the variable for degrees of freedom. So, at an invariant point, F = 0. We cannot change P OR T and stay on the point.

On a phase diagram, reaction lines intersect at invariant points. There are occasionally exceptions, discussed below, but in general this is true. Along a reaction line, we can change either P or T independently and slide up and down the line. But, once we specify a change in T, there is only one change in P that we can make and still stay on the line. So, for reaction lines, F = 1.

In open spaces, we can change both P and T independently (within limits) and still stay in the same field (field = a space between reaction lines). So, F = 2.

6. So, using the numbers you put into the previous tables, complete this the one below. You will have to logic out values for the boxes marked with question marks. You can do this by looking at the number patterns. If you do all this correctly, you have derived the phase rule.

С	P for $F = 2$	P for $F = 1$	P for $F = 0$
# components	(max) number of phases stable in a field	(max) number of phases in a reaction	(max) number of phases at an invariant point
1			
2	?		
3	?		
4	?		?

Above, I asked you to count reactions around invariant points, ut I did not ask you to look at Diagram 4. This is because many of the reactions in Diagram 4 are *degenerate*. That means they contain fewer phases than are allowed by the phase rule. This means that an invariant point may also contain fewer than the maximum number of phases allowed by the phase rule.

7. Look at Diagram 4. List two reactions that are not degenerate (in Diagram 4).

8. List two reactions that are degenerate (in Diagram 4).

Another complication with Diagram 4 is that there are *indifferent crossings* where reactions cross but do not create an invariant point.

Look at the intersection in the upper right hand corner of Diagram 4. The two reactions there involve Br, Pe, H_2O , Tr, En, Di, and Qz. This is 7 phases. Not allowed by the Phase Rule. So the intersection is NOT an invariant point.

9. Aren't you glad I did not ask many questions about Diagram 4?

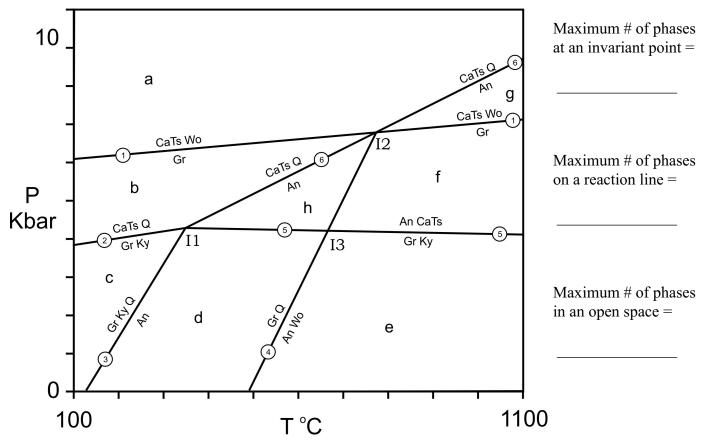
The phase diagram below includes reactions for a *system* that includes 6 minerals (listed in the table). The numbers and letters on points, lines and spaces are usually not present but are included to help you give clear answers to the questions below.

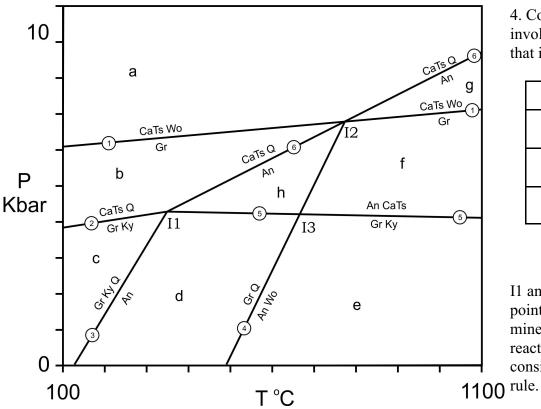
mineral	abbrev.	formula
anorthite	An	CaAl ₂ Si ₂ O ₈
CaTs (calcium tschermaks pyroxene)	CaTs	CaAl ₂ SiO ₆
grossular	Gr	$Ca_3Al_2Si_3O_{12}$
quartz	Q	SiO ₂
wollastonite	Wo	CaSiO ₃
kyanite	Ку	Al ₂ SiO ₅

1. List the oxides needed to make up any of the minerals.

2. Usually, the number of oxides is equal to the number of chemical components necessary to describe a system. So, how many components are in this system?

3. The phase rule says C + 2 = P + F where C = the number of components, P = the maximum number of phases that can coexist (in this example they are all minerals), and F = degrees of freedom (0 at an invariant point, 1 on a reaction line, and 2 in an open space). So, how many phases may coexist at an invariant point, on a line, or in an open space for the system we are talking about? (Fill in the blanks.)





4. Count the minerals involved in the reactions that intersect at I1, I2, I3.

point	# minerals
I1	
I2	
I3	

I1 and I2 are invariant points. The number of minerals involved in reactions that intersect is consistent with the phase rule.

- 5. List all the possible minerals that may coexist at I1.
- 6. List all possible minerals that may coexist at I2.

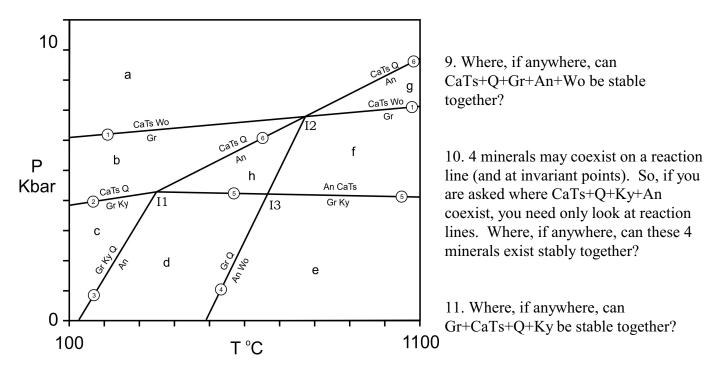
I3 is NOT an invariant point – instead it is called an "*indifferent crossing*" – because there are too many minerals in the reactions that cross there. (What this means is that reaction 4 and reaction 5 do not apply to the same composition rocks.)

Some of the reactions on this diagram involve 4 minerals. Others involve only 3. Reactions that involve fewer than the maximum allowable number of minerals are "*degenerate reactions*." Degenerate reactions involve fewer chemical components than other reactions in a system.

7. List and balance all the degenerate reactions.

8. In this system, the maximum number of minerals that can coexist is 5. And only at invariant points. So, if you are asked where CaTs+Q+Ky+An+Wo+Gr coexist, you know it is nowhere.

But, what about CaTs+Q+Ky+An+Wo (5 minerals). You need only check out the two invariant points to see if the assemblage can be. Where, if anywhere, can CaTs+Q+Ky+An+Wo+Gr be stable together?



12. 3 minerals may coexist in open spaces – perhaps more than one open space – between reaction lines (and on lines and at invariant points). So, if you are asked where CaTs+Gr+An coexist, you need only look at the spaces and the reactions that border them. Where, if anywhere, can these 3 minerals exist stably together?

13. Where, if anywhere, can Wo+Ky+CaTs be stable together? (This is a bit tricky.)

14. What about 2 minerals coexisting? Where on the diagram can Gr+Ky coexist?

15. Where on the diagram can An+Wo coexist?

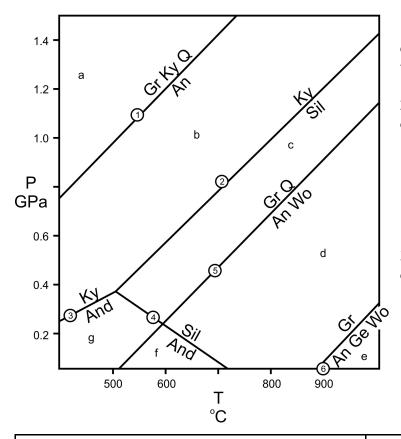
16. Where on the diagram can Gr+Wo coexist?

17. And, what about single minerals. Where on the diagram can An exist?

18. Where on the diagram can Wo exist?

19. Where on the diagram can Gr exist?

A schematic diagram involving 6 minerals is shown below. Reactions are numbered and open spaces are labeled with letters.



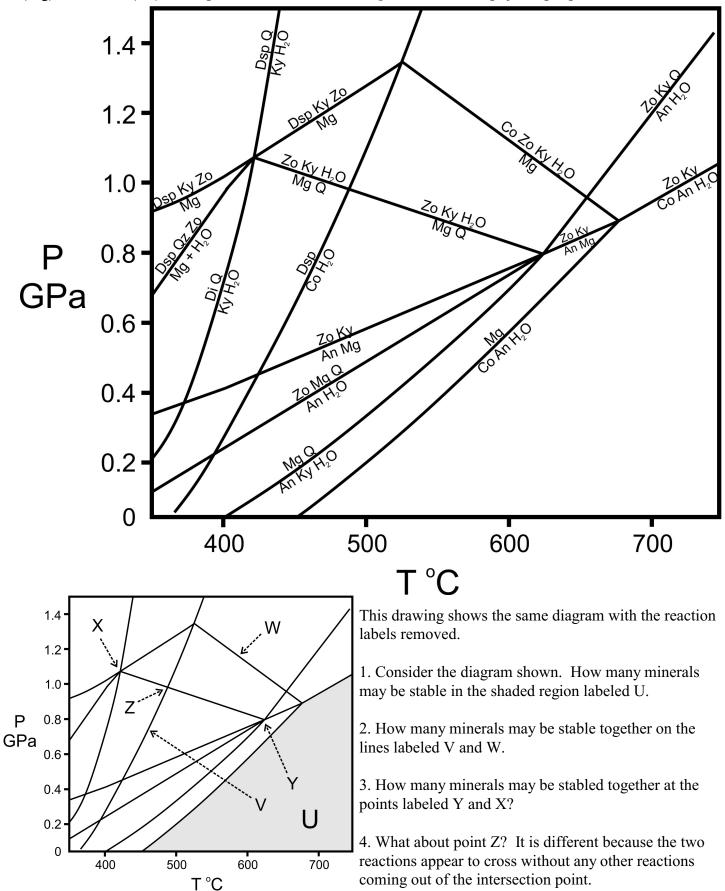
1. Consider the diagram shown. How many components are needed to describe the system that is involved?

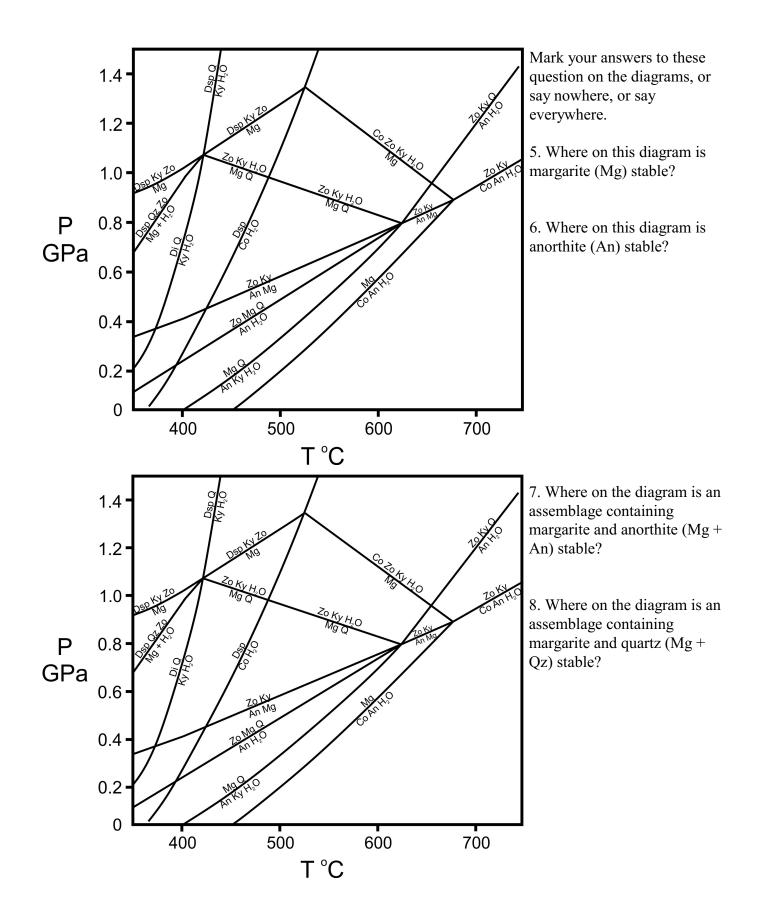
2. Which minerals are stable everywhere on this diagram?

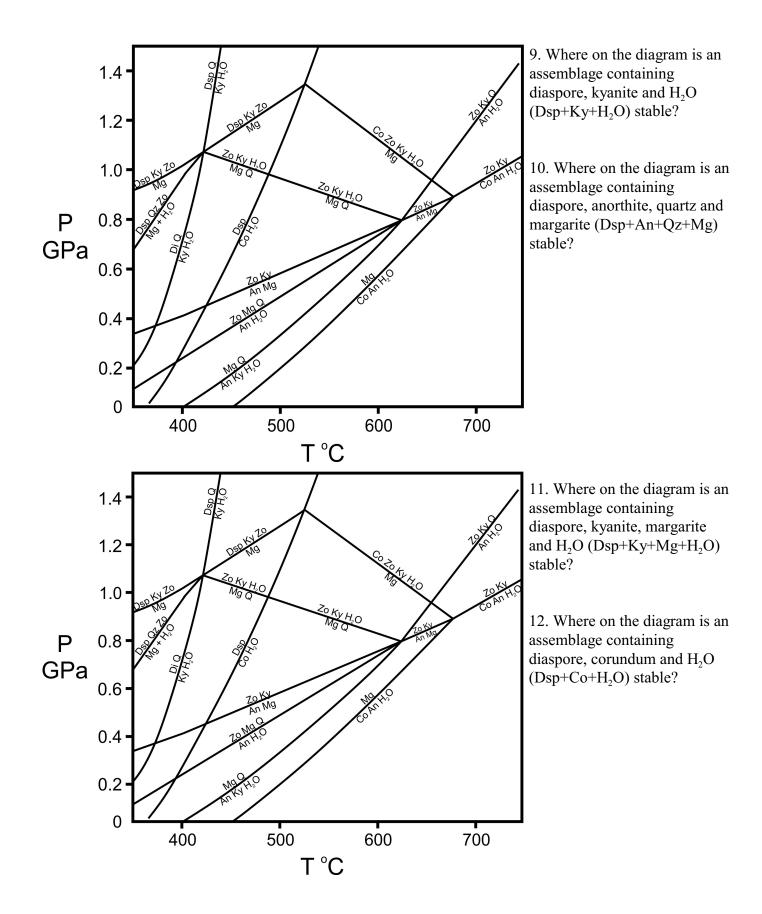
3. What about the other minerals. List them and describe where they are stable.

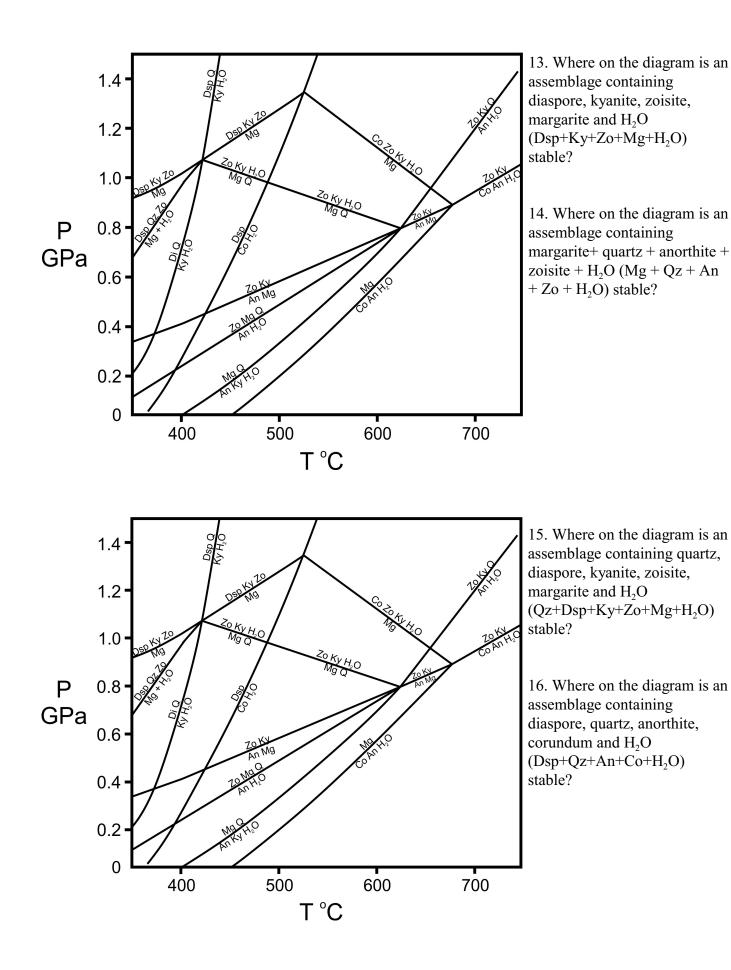
mineral/assemblage	where stable?
Sil	
Sil + Ge	
Gr + Sil	
Ky + Sil	
Ky + Q + Gr	
Sil + Gr + Q	
Ky + Q + An	
Ky + And + Sil + Gr	
An + Wo + Gr + Q	
An + Wo + Gr + And	
An + Ge + Wo + Gr + Sil	
Ky + And + Sil + Gr + Q	
Wo + Ky + And + Sil + Gr + Q	

This diagram shows reactions involving diaspore (Dsp), kyanite (Ky), zoisite (Zo), quartz (Qz), margarite (Mg), corundum (Co), and H_2O . There are 4 oxide components: CaO, Al_2O_3 , SiO_2 , H_2O .





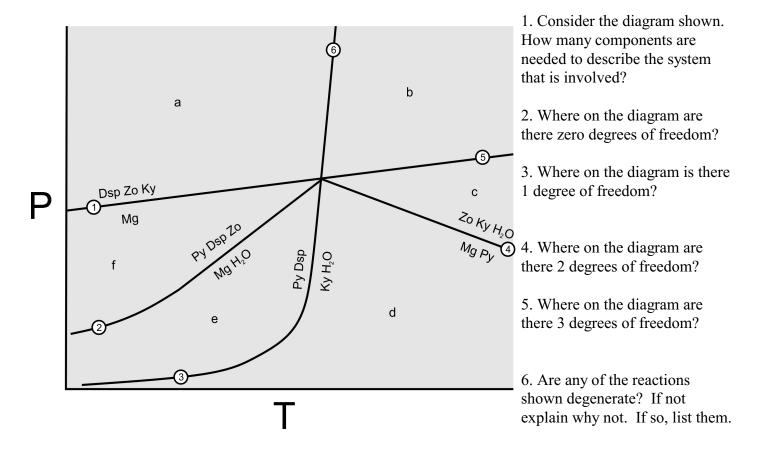




A schematic diagram involving 6 minerals is shown below. Reactions are numbered and open spaces are labeled with letters. Note that reaction 3 and reaction 6 are the same reaction on different sides of the invariant point. And reaction 1 and reaction 5 are the same reaction on different sides of the invariant point.

abbreviations:

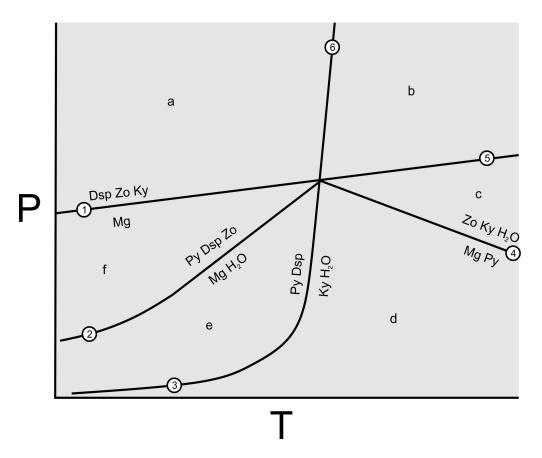
$$\begin{split} dsp &= diaspore = AlO(OH)\\ zo &= zoisite = Ca_2Al_3Si_3O_{12}(OH)\\ ky &= kyanite = Al_2SiO_5\\ Mg &= margarite = CaAl_2(Al_2Si_2)O_{10}(OH)_2\\ H2O &= water fluid \end{split}$$



7. What is the maximum number of phases that may coexist anywhere on this diagram?

8. What is the maximum number of phases that may coexist stably in an open space on this diagram?

9. Bonus question. Two of the reactions pass through the invariant point and out the other side. The other reactions do not. Explain this.



Fill in the table below to show where each of the minerals or mineral assemblages are stable.

mineral/assemblage	where stable?
Zo	
Mg	
Mg + Zo	
Dsp + Zo	
Mg + Py + Ky	
Mg + Zo + Py	
$Ky + Py + Dsp + H_2O$	
Py + Zo + Ky + Dsp	
Mg + Ky + Zo + Dsp	
$Py + Dsp + Ky + Mg + H_2O$	
Dsp + Mg + Zo + Ky + Py	
$Zo + Ky + Py + Mg + H_2O$	
$Py + Mg + Zo + Ky + H_2O$	
$Dsp + Py + Mg + Zo + Ky + H_2O$	