Deriving Darcy’s Law: Demonstration and Think-Pair-Share Exercise

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This exercise involves a demonstration of fluid flow through porous sediment using a constant head permeameter, followed by a Think-Pair-Share exercise: "What could we change in order to increase flow through the system?" The students derive Darcy's Law based upon their brainstormed list of ideas generated by the Think-Pair-Share exercise.

Prior to class:

- Set up a constant head permeameter with sediment sample, and saturate column.
  (I usually set it up with the inflow at the bottom and the outflow at the top).
- Need: 500 mL or 100 mL graduated cylinder to receive discharge.
  Stopwatch or watch that reads seconds.
  Water supply. (For classrooms without a sink, 1 or 2 liter bottles should suffice, plus a receptacle for overflow.
- Suggestion: run through the demonstration before class to work out the logistics.

Part 1: Demonstration

- Explain Henri Darcy's problem:
  Filtration of water supply using sand beds: what controls the discharge?
- And his approach to a solution:
  Experimentation to analyze the problem.

- Direct their attention to the permeameter, and ask the obvious:
  What do we have here? (brief description of the apparatus and sandy sediment).
  If the valves are opened, will water flow through it? (yes!)
  Which direction will it flow? (higher head toward lower head)

The students know intuitively that water will flow through the column, and that it will flow from the water supply at the higher elevation to the outlet at lower elevation. If the sediment column is set up with the inflow at the bottom and the discharge at the top, they will agree that *water will flow upward* through the soil column. For some, this is against their initial intuition that water only flows from higher elevation downward towards lower elevation.

- Ask for volunteers:
  1 to collect discharge with graduated cylinder and take readings,
  1 to act as the timer and record readings.
  1 to keep water supply at a constant level (if sink is not available).

- Agree on a time interval for measurements. Open the valves and have the students record the volume of water discharged at each time interval. Have them run the experiment until they have sufficient measurements to see that flow is steady.

- Define volumetric discharge (Q) if needed and calculate discharge using their measurements (Q = Volume per Time).

Part 2: Think-Pair-Share Exercise

- Return to Darcy's problem: What controls discharge? Pose the question: What could we change to increase discharge through the column? If we could change *anything* about the setup or apparatus or sample, what would increase discharge?

THINK: Have them take 1 minute on their own to brainstorm a list of ideas.

PAIR: Have them take 2 minutes to pair up with a neighbor, compare lists, and generate more ideas.

SHARE: Regroup as a class. Ask each pair to share one of their items, and continue cycling through all teams until all ideas are listed on the blackboard.
(If desired, you can use different colored chalk or separate blackboards when recording ideas related to sediment, fluid, and experimental setup).

Part 3. Develop Key Relationships

Help them sort their brainstormed ideas into the fundamentals of Darcy's Law, AND discuss their misconceptions. I have done this Think-Pair-Share several times, and they almost always come up with all of the correct relationships, and often the same misconceptions. I generally discuss the items on their brainstormed list not in the order generated, but in three basic categories: sediment, fluid, physical set up.

To increase Volumetric Discharge, change the …

Sediment:
- Use sediment having larger pore size or higher permeability.
- Sediment shape or textural maturity (e.g. rounded grains vs. angular edges).
- Sediment sorting (e.g. gravel vs. sandy-gravel).

Basic misconception: students often suggest changing to a sediment having higher porosity rather than one having larger pores. (Clay can have a very high porosity, but a very low permeability).

Fluid:
- Conduct experiment at a different temperature, since water's properties vary with temperature. Sometimes must to ask them to be specific:
    Increase the temperature or decrease it?
    Which fluid properties are significant?
    Will flow increase or decrease as density (ρ) or viscosity (μ) increases?

- Use a fluid having different density or viscosity than water: (Since the question was posed as a drinking water filtration question, they will generally not suggest this).

Part 3. (continued)

Experimental setup:
- Increase cross-sectional area,
- Increase the difference between water level at inlet and outlet,
- Use a shorter sample length.

- Can ask them about changing the orientation of the setup, for example positioning the column horizontally: Will this alter discharge?

These misconceptions recur with almost every new class:

To increase Volumetric Discharge …
… Use a larger funnel for the water supply (not a higher water level, but a wider diameter);
… Use larger diameter tubing connecting the water supply to the permeameter and/or a larger opening at the inlet and outlet;
… Use a *smaller* cross-sectional area (A) “to increase the velocity of the flow”.
… Switch inlet and outlet hoses so that flow enters at the top, exits at the bottom, and water moves downward through the sediment, with gravity rather than against it.
Note: this change in experimental setup changes dh. Need to work with them to address two scenarios: Make their suggested changes to the hoses keeping water level in the funnel (h1) at the original level vs. : Make their suggested changes to the hoses, but adjusting the position of the water supply funnel so that dh remains the same as in the original experiment.

Part 4. Explore Further (Optional)

This would be an ideal time to have them experiment with alternate setups to prove their ideas are correct or to address their misconceptions. I usually do not take the time in class, but instead have them experiment with different conditions in a follow up laboratory/homework assignment. The assignment has them use the constant head permeameter and Darcy’s Law to determine hydraulic conductivity and permeability (e.g. Lab 7 in Lee et al., 2003).

Part 5. Darcy’s Law - Putting the Key Concepts Together

Use each of their correct suggestions to develop the fact that discharge is directly or indirectly proportional to several factors:

$$Q \quad \text{depends on} \quad \begin{bmatrix}\text{sediment}\\ \text{properties}\end{bmatrix}\begin{bmatrix}\text{fluid}\\ \text{properties}\end{bmatrix}\begin{bmatrix}\text{physical setup, whether}\\ \text{lab apparatus or aquifer}\end{bmatrix}$$

$$Q \quad \propto \quad [\text{poresize, shape, sorting}]\left[\frac{\text{fluid density}}{\text{viscosity}}\right]\left[\frac{\text{cross sectional area, head difference}}{\text{length of sediment column}}\right]$$

$$Q \quad = \quad \left[C\, d^j\right] \quad \left[\frac{\rho g}{\mu}\right] \quad \left[A\, \frac{(h_{in} - h_{out})}{dl}\right]$$

where grain size (d) is used as a proxy for pore size; the relationship is non-linear, (exponent j reflects this), and coefficient C accounts for grain shape and/or sorting.

Make the following points one at a time to define terms:

- Intrinsic permeability (Ki) is a property of the porous media ($Ki = Cd^j$):

$$Q = \quad [K_i] \left[\frac{\rho g}{\mu}\right] \left[A\, \frac{(hin - hout)}{dl}\right]$$

- Hydraulic conductivity (K) depends on both the porous media and the fluid.

$$K \quad = \quad Ki \; \frac{\rho g}{\mu}$$

$$Q \quad = \quad K \quad \left[A\, \frac{(hin - hout)}{dl}\right]$$

- The hydraulic gradient is negative by convention:

$$\frac{dh}{dl} \quad = \quad \text{hydraulic gradient} \quad = \quad \frac{h_{out} - h_{in}}{dl}$$

They have now arrived at Darcy's Law:

$$Q \quad = \quad -K\, A\, \frac{dh}{dl}$$

Reference cited

Lee, K., C. W. Fetter, J. E. McCray. (2003) Lab 7: Darcy's Law and Hydraulic Conductivity, in Hydrogeology Laboratory Manual. Pearson Education, New Jersey.